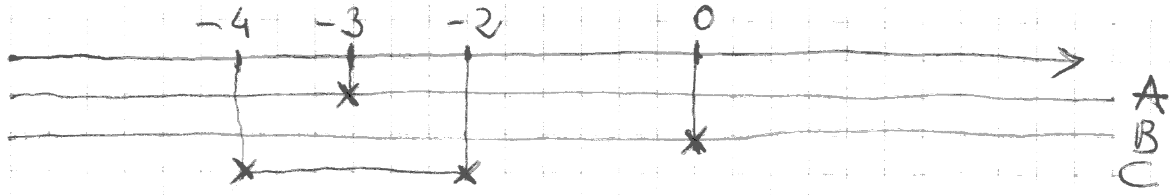
1) $A = \{x \in \mathbb{R} : x+3 \neq 0\} = \mathbb{R} \setminus \{-3\}$

$B = \{x \in \mathbb{R} : x \neq 0\} = \mathbb{R} \setminus \{0\}$

$C = (-4, -2)$




a) $C \subseteq A$? NO, perchè $-3 \in C \setminus A$

$C \subseteq B$? SI.

b) $A \cup C = \mathbb{R}$, $B \cup C = \mathbb{R} \setminus \{0\}$

c) $A \cap C = (-4,-3) \cup (-3,-2)$

$B \cap C = (-4,-2) = C$ (segue anche da $C \subseteq B$)

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2) $\dfrac{1-x^2}{x-3} - 3 \leq 0 \iff \dfrac{1-x^2-3(x-3)}{x-3} \leq 0$

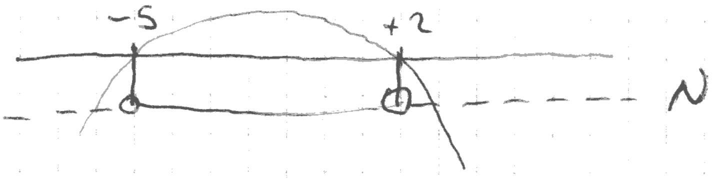
$\iff \dfrac{-x^2-3x+10}{x-3} \leq 0$

$\Rightarrow$ •) STUDIO IL SEGNO DI $-x^2-3x+10$

$-x^2-3x+10=0 \iff x = \dfrac{3 \pm \sqrt{9+40}}{-2} = \begin{cases} \dfrac{10}{-2} = -5 \\ \dfrac{-4}{-2} = +2 \end{cases}$

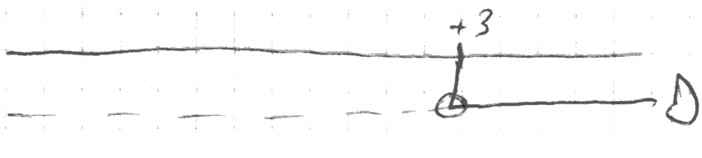
(VERIFICA: $-25+15+10=0$, $-4-6+10=0$, ok)

$\Rightarrow$

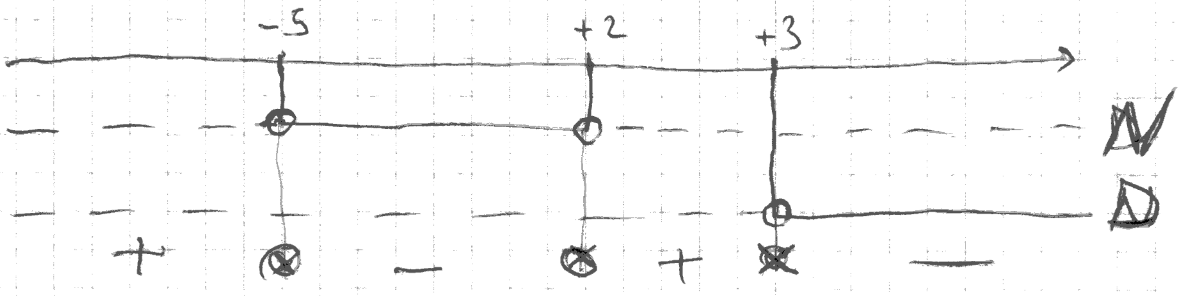



•) STUDIO IL SEGNO DI $x-3$: $x-3>0$ se $x>3$

$\Rightarrow$



$\Rightarrow$ IL SEGNO DI $\dfrac{-x^2-3x+10}{x-3}$ È:




$\Rightarrow$ LE SOLUZIONI DI $\dfrac{-x^2-3x+10}{x-3} \leq 0$ SONO:

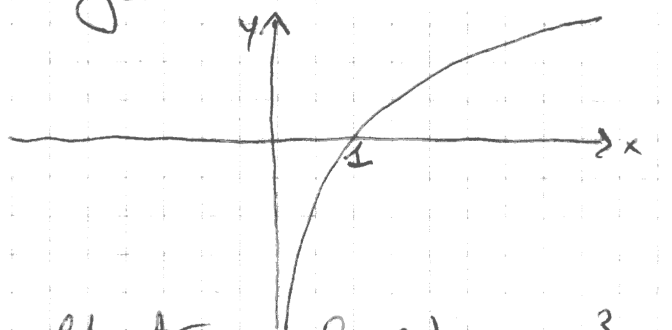
$x \in [-5, +2] \cup (+3, +\infty)$

(oppure $-5 \leq x \leq +2 \;\vee\; x > 3$)

---

3) $f(x) = \begin{cases} 1-x^3 & x<0 \rightarrow f_1 \\ \ln x & x \geq 0 \rightarrow f_2 \end{cases}$

Il grafico di $f_2(x) = \ln x$ è NOTO!



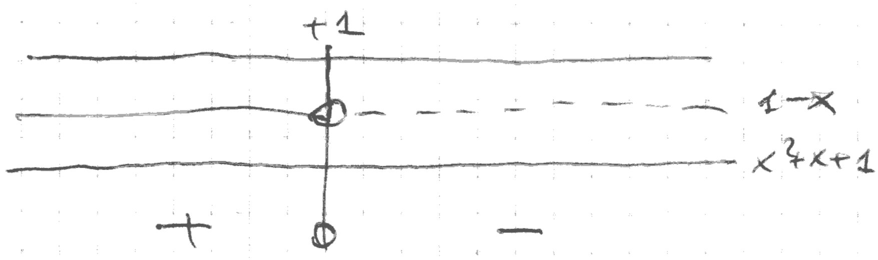

quindi studio soltanto $f_1(x) = 1-x^3$.

a) $f_1$ è definita $\forall x<0$, $f_2$ è definita $\forall x>0$

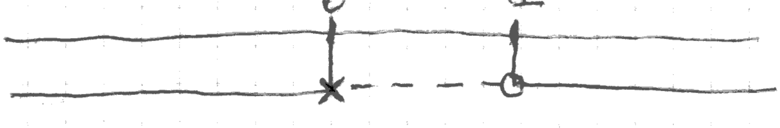
$\Rightarrow D_f = \mathbb{R} \setminus \{0\}$

PRODOTTO NOTEVOLE

b) Segno di $1-x^3 = (1-x)(x^2+x+1)$




$\Rightarrow$ Segno di $f$:



c) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} 1 - x^3 = +\infty \rightarrow \left( \lim\limits_{x \to -\infty} \frac{1-x^3}{x} = -\infty \Rightarrow \right.$ NO ASINTOTO OBLIQUO)

$\lim\limits_{x \to 0^-} f(x) = 1 - 0^3 = 1$

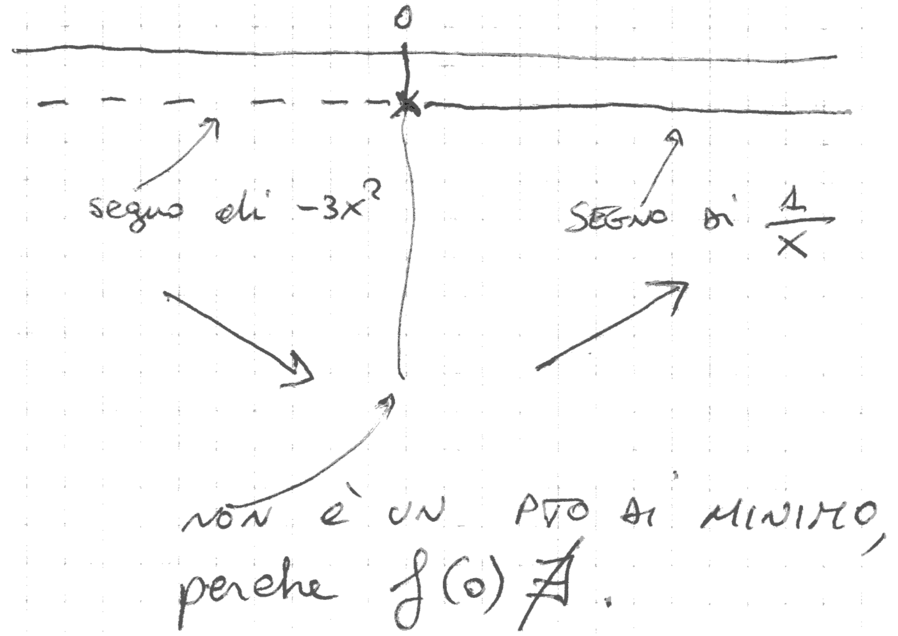
$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \ln(x) = -\infty \leftarrow$ ASINTOTO VERTICALE DX

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \ln(x) = +\infty \rightarrow \left( \lim\limits_{x \to +\infty} \frac{\ln x}{x} = 0 \Rightarrow \right.$ NO ASINTOTO OBLIQUO)

d) • $f'(x) = \begin{cases} (1-x^3)' & \text{se } x<0 \\ (\ln x)' & \text{se } x>0 \end{cases} = \begin{cases} -3x^2 & \text{se } x<0 \\ \frac{1}{x} & \text{se } x>0 \end{cases}$.

$f'(0)$ non esiste, perchè in 0 non esiste neanche $f$!

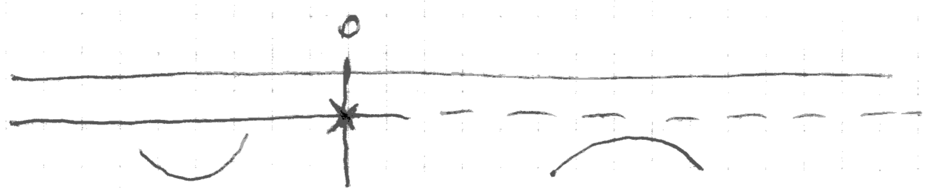
• Segno derivata:



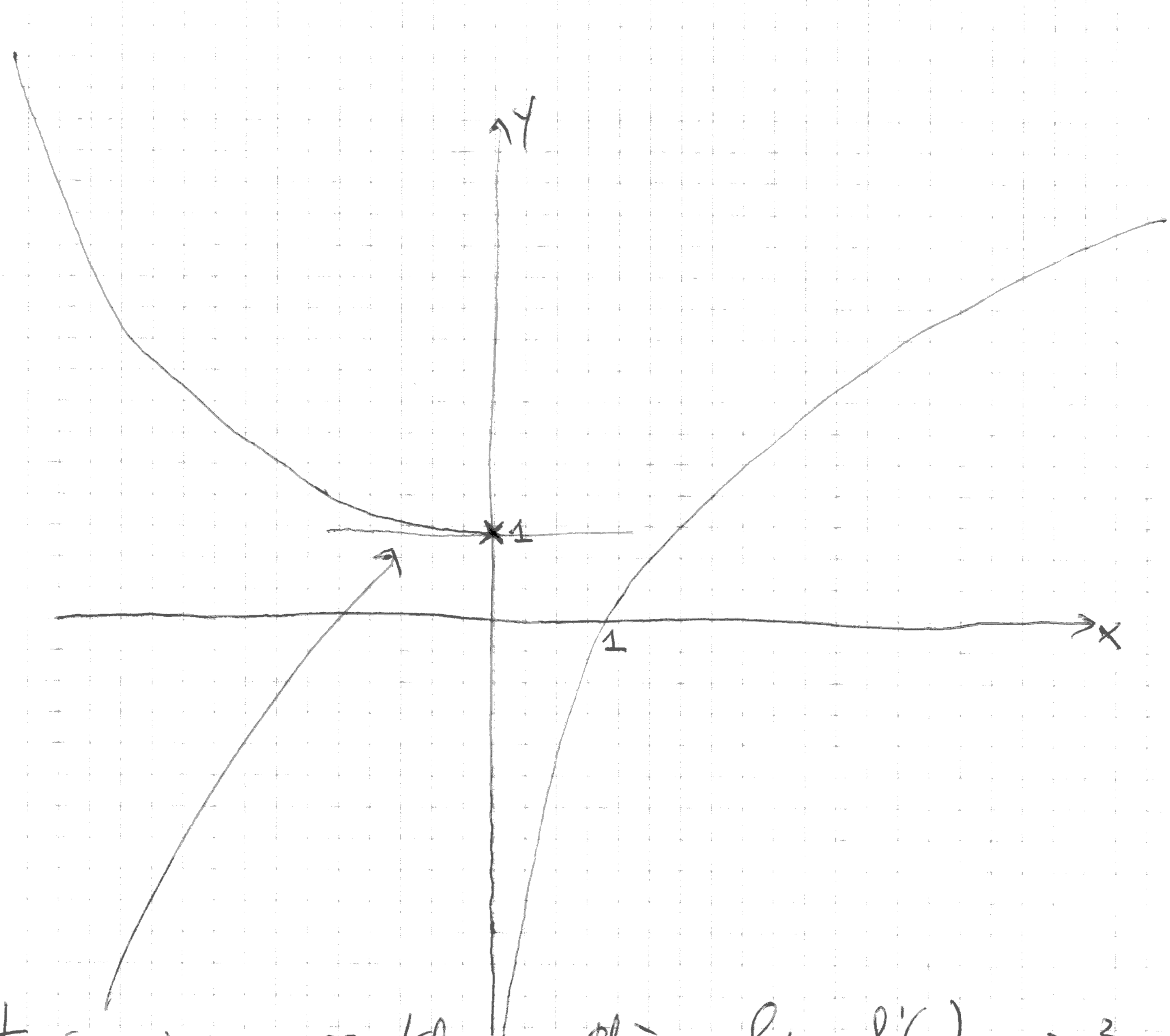

e) NON CI SONO MIN E MAX DI NESSUN TIPO.

f) $f''(x) = \begin{cases} -6x & \text{se } x<0 \\ -\frac{1}{x^2} & \text{se } x>0 \end{cases}$

$\Rightarrow$



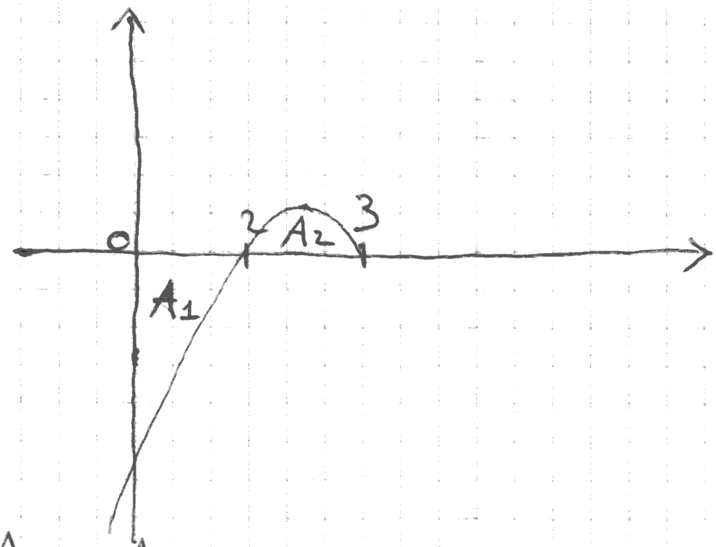
g)




tg sx in 0 orizzontale, perché $\lim_{x\to 0^-} f'(x) = -3\cdot 0^2 = 0$

4) Disegno $f(x)$ in $(0,3)$: $f(x)$ è una parabola, con intersezioni con l'asse x in +3 e +2

$\Longrightarrow$




L'area cercata è $A_1 + A_2$, con

$$A_1 = -\int_0^2 f(x)\,dx \quad , \quad e \quad A_2 = +\int_2^3 f(x)\,dx$$

$$\Rightarrow A_1 = -\int_0^2 -x^2+5x-6\,dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_0^2 =$$

$$= \frac{8}{3} - \frac{\cancel{20}^{10}}{\cancel{2}} + 12 = \frac{8}{3} + 2 = \frac{14}{3}$$

$$A_2 = +\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 6x\right]_2^3 = -\frac{27}{3} + \frac{5\cdot 9}{2} - 18 - \left(-\frac{14}{3}\right) =$$

$$= -27 + \frac{45}{2} + \frac{14}{3} = -27 + \frac{135+28}{6} = -27 + \frac{163}{6} = \frac{1}{6}$$

$$\cancel{A = A_1 + A_2 = \frac{14}{3} - 27 + \frac{163}{6} = -27 + \frac{163+28}{6} =}$$

$$\cancel{= -27 + \frac{191}{6} = \frac{-162+191}{6}}$$

$$A = A_1 + A_2 = \frac{14}{3} + \frac{1}{6} = \boxed{\frac{29}{6}}$$

5) Prima di tutto, cerco una primitiva di

$x \ln\left(1+\frac{1}{x}\right)$.

Integrando $x$ e derivando $\ln\left(1+\frac{1}{x}\right)$ viene

$$\frac{x^2}{2} \cdot \frac{1}{1+\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = -\frac{1}{2} \cdot \frac{x}{x+1} = -\frac{1}{2}\left(\frac{x+1}{x+1} - \frac{1}{x+1}\right)$$

INTEGRALE ELEMENTARE

$\Longrightarrow$ USO INTEGRAZIONE PER PARTI

$$\int x \ln\left(1+\frac{1}{x}\right) dx = \frac{x^2}{2} \ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\int\left(1 - \frac{1}{x+1}\right) dx =$$

$$= \frac{x^2}{2} \ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\left(x - \ln|x+1|\right) =$$

$$\frac{1}{2}\left(x^2 \ln\left(1+\frac{1}{x}\right) + x - \ln|x+1|\right).$$

Verifica: $$(\,''\,)' = \frac{1}{2} \cdot \left(2x \ln\left(1+\frac{1}{x}\right) + x^2 \frac{1}{1+\frac{1}{x}}\left(-\frac{1}{x^2}\right) + 1 - \frac{1}{x+1}\right) =$$

$$= \frac{1}{2}\left(2x \ln\left(1+\frac{1}{x}\right) - \frac{1}{\frac{x+1}{x}} + 1 - \frac{1}{x+1}\right)$$

$$= \frac{1}{2}\left(2x \ln\left(1+\frac{1}{x}\right) + \frac{-x + x + 1 - 1}{x+1}\right)$$

$$= x \ln\left(1+\frac{1}{x}\right) \quad \boxed{\text{ok}}$$

$$\Rightarrow \int_0^1 x \ln\left(1+\frac{1}{x}\right) dx = \lim_{t \to 0^+} \int_t^1 x \ln\left(1+1/x\right) dx =$$

$$= \lim_{t \to 0^+} \left[ \frac{1}{2}\left(x^2 \ln\left(1+\frac{1}{x}\right) + x - \ln|x+1|\right) \right]_t^1 =$$

$$= \frac{1}{2} \cdot \left(\ln 2 + 1 - \ln 2\right) - \frac{1}{2} \lim_{t \to 0^+} \left( t^2 \ln\left(1+\frac{1}{t}\right) + t - \ln|t+1| \right)$$

$$= \frac{1}{2} - \frac{1}{2} \lim_{t \to 0^+} t^2 \ln\left(1+\frac{1}{t}\right) + 0 - \ln 1 =$$

$$\frac{1}{2} - \frac{1}{2} \left( \lim_{t \to 0^+} t^2 \ln\left(1+\frac{1}{t}\right) \right) \Rightarrow$$

$$\hookrightarrow \; = \lim_{t \to 0^+} t \cdot \ln\left(1+\frac{1}{t}\right)^t =$$

$$= 0 \cdot \ln e = 0$$

$$\Rightarrow \boxed{\int_0^1 x \ln\left(1+\frac{1}{x}\right) dx = \frac{1}{2}}$$

6) a) $\alpha = 2 \Rightarrow \sum_{n=3}^{+\infty} \left(\frac{3}{5}\right)^{n-2} \Rightarrow$ converge

$\alpha = -3 \Rightarrow \sum \left(\frac{-2}{5}\right)^{n-2} \Rightarrow$ converge

$\alpha = 7 \Rightarrow \sum \left(\frac{8}{5}\right)^{n-2} \Rightarrow$ diverge

$\alpha = -8 \Rightarrow \sum \left(\frac{-7}{5}\right)^{n-2} \Rightarrow$ INDETERMINATA

b) La serie geometrica $\sum q^n$ converge se $-1 < q < 1$, diverge se $q \geq 1$, è INDETERMINATA se $q \leq -1 \Rightarrow$

$\Rightarrow \sum \left(\frac{\alpha+1}{5}\right)^{n-2}$ sarà:

$$\begin{cases} \text{CONVERGENTE (C)} & \text{se } -1 < \frac{\alpha+1}{5} < 1 \\ \text{DIVERGENTE (D)} & \text{se } \frac{\alpha+1}{5} \geq 1 \\ \text{INDETERMINATA (I)} & \text{se } \frac{\alpha+1}{5} \leq -1 \end{cases} \Longleftrightarrow$$

$$\begin{cases} C & \text{se } -6 < \alpha < 4 \\ D & \text{se } \alpha \geq 4 \\ I & \text{se } \alpha \leq -6 \end{cases}$$

7) a) $A$ ha una colonna di 0, quindi ~~d~~ $\det A = 0$

$\Rightarrow A$ non è invertibile.

b) $$A \cdot B = \begin{pmatrix} \cancel{-1\ 2\ 9} & +1 \\ & -1 \\ & -9 \end{pmatrix}$$

c) $\operatorname{Det} A = 0 \Rightarrow rg A < 3$. Vediamo se $rg A = 2$.

$$\det \begin{pmatrix} 294 & -1 \\ -437 & +1 \end{pmatrix} = 294 + 437 \neq 0 \Rightarrow rg A = 2$$

$rg B = 1$ perché ~~ha~~ $\det(2) \neq 0$