The 5th Workshop "Complex Geometry and Lie Groups"

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Abstract

[Plenary Talks]

[A] **Daniele Angella**

"Isometric immersions of locally conformally Kähler manifolds"

The celebrated Kodaira Embedding Theorem gives geometric and cohomological con- ditions under which analytic geometry reduces to algebraic geometry. In general, such an embedding is not isometric. The problem of which real-analytic Kähler manifolds admit an isometric immersion into $\mathbf{CP}^n$, or more in general into complex space forms has been studied by Eugenio Calabi. Locally conformally geometry can be intepreted as a sort of "equivariant (homo- thetic) Kähler geometry" and a first specific non-Kähler setting. Despite the Kähler condition imposes strong topological obstructions, most of the known compact complex surfaces admit locally conformally Kähler structures. In the lcK context, the analogue of the projective space is played by Hopf manifolds, and an analogue of the Kodaira embedding has been proven by Liviu Ornea and Misha Verbitsky. Inspired by Eugenio Calabi ' s work, we study isometric immersions of lcK manifolds into Hopf manifolds. In particular, we focus on non-Kähler compact complex surfaces. We also discuss problems concerning cohomological invariants of locally conformally Kähler or symplectic manifolds, and classifications of lcs and lcK structures on Lie groups. The talk is based on a joint work with Michela Zedda, and on works with Adrián Andrada, Giovanni Bazzoni, Marcos Origlia, Alexandra Otiman, Maurizio Parton, Nico- letta Tardini, Luis Ugarte.

[C] **Vicente Cortés**

"Quaternionic Kähler manifolds of co-homogeneity one"

Quaternionic Kähler manifolds form an important class of Riemannian manifolds of special holonomy. They provide examples of Einstein manifolds of non-zero scalar curvature. I will show how to construct explicit examples of complete quaternionic Kähler manifolds of negative scalar curvature beyond homogeneous spaces. In particular, I will present examples of co-homogeneity one in all dimensions, based on joint work with Dyckmanns, Jüngling and Lindemann, see arXiv:1701.07882 [math.DG].

[D] **Josef Dorfmeister**

"Surface classes, generalized Gauss maps, and harmonic maps"

There are a number of examples of surface classes discussed in the literature, for which one knows a certain "generalized Gauss map" into some k-symmetric space such that this generalized Gauss map is harmonic if and only if the original surface is of a very specific type. The classical example consists of all immersions into $\mathbf{R}^3$ with "generalized Gauss map" = "classical Gauss map" into $\mathbf{S}^2$. In this case, by a result of Ruh-Vilms, the immersion is of constant mean curvature if and only if the Gauss map is harmonic. Actually, by a result of Kenmotsu, any harmonic map from a surface into $\mathbf{S}^2$ is the Gauss map of some surface of constant mean curvature into $\mathbf{R}^3$. We will discuss several examples of this type. One of these examples will be conformal Willmore surfaces into spheres. In this case, one considers the class of conformal immersions into spheres and as generalized Gauss map their "conformal Gauss map". Then, by a result of Blaschke, Bryant, Eijiri, Rigoli, the conformal Gauss map of a conformal immersion is harmonic if and only if the immersion is a Willmore surface. This is the Ruh-Vilms type of result for Willmore surfaces into spheres. The Kenmotsu type result fails for several reasons here. But under additional assumptions it still holds. In this talk we will discuss the situation described above.

[G] **Martin Guest**

"The enhanced Coxeter Plane - an application of integrable systems to Lie groups"

The Dynkin Diagram and the Stiefel Diagram are "visualizations" of Lie groups (or their root systems) which are very familiar to geometers and topologists. We propose an "enhanced" Coxeter Plane as a third visualization. This has a purely Lie-theoretic definition, but it arises most naturally from a certain isomonodromic family of meromorphic connections, constructed as a special case of the Toda equations - an integrable system. This is joint work with Nan-Kuo Ho.

[H] **Jun-Muk Hwang**

"Rigidity of Legendrian singularities"

Let (M, D) be a holomorphic contact manifold, i.e., a complex manifold M of dimension 2m+1 equipped with a holomorphic contact structure D. An m-dimensional complex analytic subvariety V in M is called a Legendrian subvariety if the smooth locus of V is tangent to D. A Legendrian singularity means the germ of a Legendrian subvariety at a point. We discuss some rigidity results on Legendrian singularities and their relation with the geometry of Fano contact manifolds.

[K] **Ryoichi Kobayashi**

"Holomorphic Curves in Compact Complex Parallelizable Manifold $\Gamma\backslash \mathrm{SL}(2,\mathbf{C})$"

Let $\Gamma \subset \mathrm{SL}(2,\mathbf{C})$ be a cocompact lattice and $X = \Gamma\backslash \mathrm{SL}(2,\mathbf{C})$ the associated compact complex parallelizable manifold. We show that any non-constant holomorphic map $f : M \to X$ from a compact Riemann surface M into a compact complex parallelizable manifold X is expressed as a composition $f = t \circ h \circ \alpha$ where $\alpha : M \to \mathrm{Alb}(M)$ is the Albanese map, the map $h : \mathrm{Alb}(M) \to X = \Gamma\backslash \mathrm{SL}(2,\mathbf{C})$ has its image in a maximal torus $T = \Gamma \cap A\backslash A \cong \mathbb{Z}\backslash \mathbf{C}^*$ in X defining an algebraic group homomorphism $h : \mathrm{Alb}(M) \to T = A \cap \Gamma\backslash A$, and finally t is a right translation by some element of $\mathrm{SL}(2,\mathbf{C})$. The proof is based on Bishop's criterion of analyticity of sets and a simple observation in hyperbolic geometry.

[M] **Takuro Mochizuki**

"Periodic monopoles and difference modules"

One of the main themes in complex geometry is to obtain a correspondence between differential geometric objects and algebro-geometric objects. For instance, it is notable that Simpson proved the equivalence of irreducible tame harmonic bundles, stable parabolic bundles with logarithmic connections and stable parabolic bundles with logarithmic Higgs fields on compact punctured Riemann surfaces.

In this talk, we shall explain an equivalence between singular periodic monopoles of GCK type and stable parabolic difference modules. It is a variant of Simpson's theorem in the context of periodic monopoles.

[N] **Yoshinori Namikawa**

"Symplectic singularities and nilpotent orbits"

After introducing the finiteness theorem for symplectic singularities, I will give a characterisation of nilpotent orbit closures of a complex semisimple Lie algebra.

[O] **Yoshihiro Ohnita**

"Lagrangian Geometry of the Gauss images of isoparametric hypersurfaces in spheres"

The images of the Gauss map (Gauss images) of isoparametric hypersurfaces in the standard sphere provide a nice class of compact minimal (thus monotone) Lagrangian submanifolds embedded in a complex hyperquadrics which is a compact Hermitian symmetric space of rank 2. It is an interesting problem to study the properties of such Lagrangian submanifolds by the well-developed theory of isoparametric hypersurfaces. In this talk I will give a survey of our recent works and environs on Hamiltonian stability problem, Hamiltonian non-displaceability and Floer homology of such Lagrangian submanifolds. This talk is based on my joint works on Hui Ma (Tsinghua University, P.R. China), Hiroshi Iriyeh (Ibaraki University, Japan) and Reiko Miyaoka (Tohoku University, Japan).

[Pc] **Tommaso Pacini**

"The complex Lie group of totally real submanifolds"

A "totally real" submanifold is the exact opposite of a complex submanifold. Classically, such submanifolds were of interest mostly in holomorphic function theory, but recent work has emphasized that there exists an interesting "totally real geometry". In particular, the space of totally real submanifolds can be described as a (infinite-dimensional) complexified Lie group: this provides an interesting framework for the study of minimal Lagrangian submanifolds. I will give an overview of the subject, largely based on joint work with J. Lotay (UCL) and on work by R. Maccheroni (Univ. of Parma).

[Pn] **Yat Sun Poon**

"Schouten Algebra of Holomorphic Poisson Cohomology on Nilmanifolds"

On nilmanifolds with abelian complex structures, we identify a type of holomorphic Poisson structures for which its associated holomorphic Poisson cohomology is isomorphic to Dolbeault cohomology as Schouten algebra.

[Sk] **Yusuke Sakane**

"Invariant Einstein metrics on Stiefel manifolds"

The study of finding invariant Einstein metrics on Stiefel manifolds was originally started by S. Kobayashi, A. Sagle and G. Jensen. Then Arvanitoyeorgos, Dzhepko and Nikonorov obtained new invariant Einstein metrics on the Stiefel manifolds $V_{sk}\mathbb{R}^{sk+\ell} \cong SO(sk+\ell)/SO(\ell)$ by making some extra symmetry assumptions in 2006. Moreover, by improving their results, Arvanitoyeorgos, Sakane and Statha obtained more Einstein metrics on the

Stiefel manifolds in 2013. The difficulty to find invariant Einstein metrics on Stiefel manifolds is that the description of Ricci tensors for general invariant metrics are not easy, since their isotropy representation contains equivalent summands. In the present work we show existence of new invariant Einstein metrics on the Stiefel manifolds $V_{n-2p}\mathbb{R}^n$ ($n \geq 9$ and $2 \leq p \leq (2/5)n - 1$). Our method is based on considering Stiefel manifolds as total spaces over generalized flag manifolds with two isotropy summunds and solving the algebraic quations by computing Gröebner basis.

[Sm] **Uwe Semmelmann**

"Killing tensors on Riemannian manifolds"

Killing tensors are symmetric tensors such that the complete symmetrization of the covariant derivative vanishes. This generalizes the equation for Killing vector fields. Killing tensors are well studied in physics, in particular since they define first integrals, i.e. functions constant on geodesics.
In my talk I will introduce a formalism for studying Killing and conformal Killing tensors. Using this notation I will discuss the most important properties and mention a few recent results, e.g. the non-existence on compact manifolds with negative sectional curvature and a classification result on Riemannian products. Moreover I will describe several examples of Killing tensors. My talk is based on two joint articles with K. Heil and A. Moroianu.

[U] **Luis Ugarte**

"The Hull-Strominger system and holomorphic deformations"

Strominger and Hull investigated independently the heterotic superstring background with non-zero torsion, which led to a complicated system of partial differential equations. Roughly speaking, and in six dimensions, the system requires a compact complex conformally balanced manifold X with holomorphically trivial canonical bundle, equipped with an instanton compatible with the Green-Schwarz anomaly cancellation equation for a non-zero constant α', which is the slope parameter in string theory. Fu and Yau proved the existence of solutions on non-Kähler Calabi-Yau manifolds given as a $\mathbb{T}^2$-bundle over a $K3$ surface. Explicit solutions of the Hull-Strominger system have been constructed by several authors on compact quotients of 6-dimensional nilpotent or solvable Lie groups and of SL(2, $\mathbb{C}$). A compact complex manifold X is said to have the Hull-Strominger property if it admits a solution to the previous system. In this talk we will show that this property is not stable under small deformations of the complex structure. For the construction of the deformations, we first consider the balanced Hermitian geometry on the nilmanifold underlying the Iwasawa manifold, and then make use of some specific complex structures for which their Bott-Chern cohomology group of bidegree (2,2) satisfies a special condition. Joint work with Stefan Ivanov.

[V] **Luigi Verdiani**

"Invariant metrics on cohomogeneity one Riemannian manifolds"

A cohomogeneity one Riemannian manifold M is a smooth Riemannian manifold, acted on by a subgroup G of the isometry group with a codimension one orbit. In many practical problems, the high degree of symmetry of M can be used to reduce certain PDE's to ODE's that are, in principle, easier to study. In fact this class has been used to produce many inhomogeneous examples of special Riemannian structures. The existence of a codimension one orbit in M implies that the union of the codimension one orbits is an open dense set in M. If M is compact and simply connected, then there are 2 orbits of higher codimension and, because of this, it is not trivial to describe all the smooth G- invariant metrics on M. During the talk we will discuss this problem and some applications to the local existence of Einstein metrics and existence of metrics with prescribed Ricci tensor on M.

[Research Talks]

[A] **Takao Akahori**

"CR invariants and Hamiltonian flows"

By a successsful work of deformation theory of CR structures, the notion of CR Hamiltonian flows is found. In this work, we show that some CR invariants do not change along this flow.

[Bl] **Florin Belgun**

"Convexity for locally conformally symplectic manifolds"

We obtain an ananalogue of the classical Atiyah-Guillemin-Sternberg Theorem in symplectic geometry, more precisely we show that a twisted Hamiltonian action of Lee type of a torus on a compact locally conformally symplectic manifold has a convex tiwtsed moment map and describe its image.

[Br] **Aleksandra Borowka**

"Cones and generalized Feix-Kaledin construction"

Let M be a $4n$-dimensional quaternionic manifold equipped with a quaternionic S^1 action with no triholomorphic points and let S be the fixed point set of the action. Suppose that S has the maximal possible dimension, i.e., $2n$. Then S inherits from the quaternionic structure on M an integrable complex structure and the class of quaternionic connections induces a c-projective structure on S. The generalized Feix–Kaledin construction provides a way (using twistor methods) to construct any such M from S, assuming that S is equpped with a real-alnalytic c-projective structure with type $(1,1)$ Weyl curvature and with a holomorphic line bundle with a connection of type $(1,1)$ curvature. Note that introducing the twist by a line bundle allows to construct many quaternionic manifolds from a fixed c-projective structure on S. In particular from the c-projective structure containing the Levi-Civita connection of the Fubini-Study metric, we can construct both hyperkahler Calabi structure and the quaternion-Kahler Grassmannian $Gr_2(\mathbb{C}^{n+2})$. In this talk we will focus on quaternion-Kahler case of the construction. To achieve this we will discuss the relation of the generalized Feix–Kaledin construction with the Swann bundle and Armstrong cone constructions.

[G] **Rebecca Glover**

"Lagrangian-type submanifolds of $G2$ and $Spin(7)$ manifolds"

The study of Lagrangian submanifolds has played a fundamental role in furthering the field of symplectic geometry. Lagrangian submanifolds reveal information about Hamiltonian mechanics, symplectic rigidity, and local invariants of symplectic manifolds. Further, a deeper understanding of Lagrangian submanifolds has provided insight towards establishing a correspondence between Calabi-Yau mirror pairs in Kontsevich's homological mirror symmetry via the Fukaya category. In this talk, we discuss the analogues for Lagrangian submanifolds in G2 and Spin(7) geometry. We will discuss properties of these submanifolds as well as their deformation spaces. We will also discuss a complex-like structure induced on one of these types of submanifolds. This is joint work with Sema Salur.

[H] **Yoshinori Hashimoto**

"Twisted constant scalar curvature Kähler metrics and a continuity method in Kähler geometry"

Whether a constant scalar curvature Kähler (cscK) metric exists on a Kähler manifold is a question that attracted much attention in recent years, particularly in terms of the conjectured relationship with stability notions in Geometric Invariant Theory. A continuity method for finding such metrics was proposed by X.X. Chen, which involves a "twisted" cscK metric that is an interesting object in its own right. We prove openness for this continuity method, and quickly survey some recent relevant results.

[K] **Hisashi Kasuya**

"Techniques of constructions of variations of mixed Hodge structures"

The purpose of this talk is to give a way of constructing real variations of mixed Hodge structures (R-VMHS) over compact Kahler manifolds by using mixed Hodge structures on Sullivan ' s 1-minimal models. This construction is very similar to known ideas (e.g Hain-Zucker). But this may be essentially different from them, since the obtained R-VMHS depends on Kahler metric. It is expected that this construction gives a differential geometric view of R-VMHSs.

[N] **Satoshi Nakamura**

"Generalized Kähler Einstein metrics and uniform stability for toric Fano manifolds"

The existence problem of Kähler Einstein metrics for Fano manifolds was one of the central problems in Kähler Geometry. The vanishing of the Futaki invariant is known as an obstruction to the existence of Kähler Einstein metrics. Generalized Kähler Einstein metrics, introduced by Mabuchi in 2000, is a generalization of Kähler Einstein metrics for Fano manifolds with non-vanishing Futaki invariant. In this talk, we give a complete criterion for the existence of generalized Kähler Einstein metrics on toric Fano manifolds from view points of a uniform stability in a sense of GIT (geometric invariant theory) and the properness of a functional on the space of Kähler metrics. Since our criterion is based on the general framework of GIT and Kähler geometry, these lines to attack the existence problem of generalized Kähler Einstein metrics are expected to extend for general Fano manifolds. This talk is based on arXiv:1706.01608 (to appear in Tohoku Math J.).

[Or] **Marcos Origlia**

"Vaisman structures on solvmanifolds"

The most important class of Hermitian manifolds are the well known Kähler manifolds. Another class, much studied, is given by the *locally conformally Kähler* (LCK) manifolds, that is, Hermitian manifolds whose metric is conformal to a Kähler metric in some

neighbourhood of each point. Among them, there is a outstanding class called Vaisman manifolds which are very interesting because of its topological properties and relations with other geometric structures. In this talk we study Vaisman structures on compact quotients $\Gamma\backslash G$ of a simply connected solvable Lie group G by a lattice Γ, where these structures come from left-invariant Vaisman structures on G, or equivalently, from Vaisman structures on the Lie algebra of G.
We characterize unimodular solvable Lie algebras with Vaisman structures in terms of Kähler flat Lie algebras and a suitable derivation. Applying this result we get obstructions for the existence of such structures on a Lie algebra. Using this characterization we build families of Lie algebras and Lie groups admitting a Vaisman structure and show the existence of lattices in some of these groups.
Finally, we also determine algebraic restrictions that the Lie algebra associated to a solvmanifold admitting a Vaisman structure must satisfy in order to guarantee that the canonical bundle of this complex solvmanifold is holomorphically trivial.

[Ot] **Alexandra Otiman**

"Cohomology of Oeljeklaus-Toma manifolds"

Oeljeklaus-Toma manifolds are a higher-dimensional generalization of Inoue-Bombieri surfaces and were introduced by K. Oeljeklaus and M. Toma in 2005. They are quotients of $\mathbb{H}^s \times \mathbb{C}^t$ by discrete groups of affine transformations arising from a number field K and a particular choice of a subgroup of units U of K. They are commonly referred to as OT manifolds of type (s, t). OT manifolds have been of particular interest for locally conformally Kähler (lcK) geometry since they do not admit Kähler metrics, but those of type $(s, 1)$ admit lcK metrics and for (s, t) in general, the existence of an lcK metric reduces to a number-theoretic condition. In this talk, we compute their de Rham and twisted cohomology and derive several applications concerning the characterization of possible Lee classes for lcK metrics, the twisted cohomology class of lcK metrics and the low-degree Chern classes of complex vector bundles over OT manifolds.

[R] **Federico A. Rossi**

"Construction of nice nilpotent Lie algebras and Einstein metrics"

Nice nilpotent Lie algebras play an important role in the study of Ricci flow and in the construction of nilsolitons, as well as they can carry invariant Einstein pseudo-Riemannian metrics. However, to our best knowledge, there is a classification of such structures only for low dimension, and a construction theory is missing. In this talk we will present an algorithm to obtain nice nilpotent algebras, and we will also show that the these nice Lie algebras are different in a suitable category. Moreover, we will give some examples and a constructive method to build special Einstein pseudo-Riemannian metrics, proving that there are some obstructions to their existence. Time permitting, we will discuss some links between our results and nilsolitons. This is a joint work with Diego Conti

[Td] **Homare Tadano**

"Myers-Type Theorems, Diameter Bounds, and Gap Theorems for Sasaki Manifolds"

After reviewing basic facts about Sasaki geometry, we shall give some Myers-type theorems for complete Sasaki manifolds in the spirit of Cheeger-Gromov-Taylor (J. Diff. Geom. 17 (1982)). Moreover, we shall give upper and lower diameter bounds for compact gradient

Sasaki-Ricci solitons in terms of the transverse scalar curvature. Finally, we shall give some gap theorems for compact gradient Sasaki-Ricci solitons, showing some necessary and sufficient conditions for the solitons to be Sasaki-Einstein.

[Tk] **Ryosuke Takahashi**

"The inverse Monge-Ampère flow and applications to Kähler-Einstein metrics"

We introduce the "inverse Monge-Ampère flow", a new parabolic flow which is designed to deform a given Kähler metric to a Kähler-Einstein one, and fits Donaldson's new GIT picture. We provide some convergence results for the inverse Monge-Ampère flow. This talk is based on a joint work with T. C. Collins (Harvard Univ.) and T. Hisamoto (Nagoya Univ.).

[Tr] **Nicoletta Tardini**

"Cohomology groups on complex non-Kähler manifolds"

The $\partial\bar{\partial}$-lemma represents an important obstruction to Kählerianity on compact complex manifolds. This property is cohomological and it is strongly related to the Bott-Chern and the Aeppli cohomology groups. We will de- scribe this relation and then we will discuss the behavior of the $\partial\bar{\partial}$-lemma under natural operations on complex manifolds. This is a joint work with Daniele Angella, Tatsuo Suwa and Adriano Tomassini.