The Complexity of Model Checking in Modal Event Calculi

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Kowalski and Sergot's *Event Calculus* (*EC*) is a simple temporal formalism designed to model situations characterized by a set of *events* whose occurrences have the effect of starting or terminating the validity of determined *properties*. *EC* is able to determine the *maximal validity intervals* (*MVIs*) over which a property holds uninterruptedly. The algorithm *EC* relies on for the verification of MVIs (a model checking problem) is polynomial. It can advantageously be implemented as a logic program.

In situations consisting of a fixed set of event occurrences but incomplete information about their relative order, the *Modal Event Calculus* (*MEC*) extends *EC* with the possibility of inquiring in polynomial time about intervals which will remain MVIs no matter what new ordering information is acquired ($\Box$-*MVIs*) and intervals that are MVIs in some completion of the current event ordering ($\Diamond$-*MVIs*). The *Generalized Modal Event Calculus* (*GMEC*) enhances the expressive power of *MEC* by supporting a free mixing of boolean connectives and modalities, but at the cost of intractability.

ECMEC and *ICMEC* are intermediate calculi between *MEC* and *GMEC*. The former allows only combining computations of MVIs, $\Box$-MVIs and $\Diamond$-MVIs by means of boolean connectives. Dually, the latter only permits boolean combinations of MVI computations to be prefixed by either $\Box$ or $\Diamond$. These calculi are strictly more expressive than *MEC*, but, while model checking in *ECMEC* is still polynomial, it is NP-hard in *ICMEC*.

The following table summarizes the cost of model checking in these calculi as a function of the number of events and the number of atomic formulas:

Calculus	*EC*	*MEC*	*ECMEC*	*ICMEC*	*GMEC*
Parameters	n events	n events	n events k atomic formulas	n events k atomic formulas	n events k atomic formulas
Model checking	$O(n^3)$	$O(n^3)$	$O(kn^3)$	NP-hard	NP-hard