Verification of Programs by Combining Iterated Specialization with Interpolation

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Motivations: Proving Partial Correctness

Given the program increment and the specification \( \varphi \)

\[
\text{while}(\ast) \{
\text{x} = \text{x} + \text{y}; \\
\text{y} = \text{y} + 1;
\}
\]

\{x=1 \land y=0\} increment \{x \geq y\}

(A) generate the verification conditions (VCs)

1. \( x=1 \land y=0 \rightarrow P(x, y) \) Initialization
2. \( P(x, y) \rightarrow P(x + y, y + 1) \) Loop invariant
3. \( P(x, y) \rightarrow x \geq y \) Exit

(B) prove they are satisfiable

If satisfiable then the Hoare triple holds.
Constraint Logic Programming (CLP) is a metalanguage for representing programs and their semantics, properties and their proof rules, i.e., for representing the Verification Conditions (VCs)

1. $x=1 \land y=0 \rightarrow P(x, y)$
2. $P(x, y) \rightarrow P(x + y, y + 1)$
3. $P(x, y) \rightarrow x \geq y$

The VCs are encoded as a constraint logic program $V$:

1. $p(X, Y) :- X=1, Y=0.$
2. $p(X1, Y1) :- X1=X+Y, Y1=Y+1, p(X, Y).$
3. $\text{unsafe} :- Y>X, p(X, Y).$

The VCs are satisfiable iff $\text{unsafe}$ not in the least model $M$ of $V$. 

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Motivations

- **Constraint Logic Programming** (CLP) is a metalanguage for representing *programs* and their *semantics*, *properties* and their *proof rules*
  i.e., for representing the Verification Conditions (VCs)

**Methods for proving the satisfiability of CLP/CHC VCs:**
- CounterExample Guided Abstraction Refinement (CEGAR), Interpolation, Satisfiability Modulo Theories
  [Bjørner et al., Duck et al., Rybalchenko et al., Rümmer et al.]
- Symbolic execution of CLP
  [Jaffar et al.]
- Static Analysis and Transformation of CLP
  [Gallagher et al., Albert et al., Fioravanti et al.]
**Motivations**

- **Constraint Logic Programming (CLP)** is a metalanguage for representing
  - *programs* and their *semantics*,
  - *properties* and their *proof rules*
  i.e., for representing the Verification Conditions (VCs)

- **Program Transformation** is a technique that
  - changes the *syntax* of a program,
  - preserves its *semantics*
  i.e., for passing information between Solvers

\[
\text{prop} \in M(\text{VCs}) \iff \text{prop} \in M(\text{VCs}^T)
\]
Verification method for program safety that combines

- Program Specialization
  - unfold/fold transformations + widening
- Interpolating Horn Clause (IHC) solving
  - top-down evaluation + interpolation

by exploiting the common Horn Clause representation of the problem. Hence, combining the effect of interpolation to the effect of widening.

Specialization and Interpolation phases can be:

- iterated, and also
- combined with other transformations that change the direction of propagation of the constraints:
  - forward from the program preconditions, or
  - backward from the error conditions.
Program transformation of **Constraint Logic Programs** (CLP) to:

- **generate** the Verification Conditions (VCs)
- **prove** the satisfiability of the VCs

**Verification Conditions: VCs**

1. Encode into CLP
2. Specialize Int w.r.t. prog
   - (generate the VCs)
3. Propagate \( \varphi_{init} \) or \( \varphi_{error} \) and Analyze
   - (prove the satisfiability of the VCs)

**Verification method:** (1); (2); (3)\(^+\)

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Given the specification \( \{ \varphi_{init} \} \ prog \ \{ \neg \varphi_{error} \} \)

**Definition (The interpreter \( Int \))**

- `unsafe :- initConf(X), reach(X).`  \( \mid X \) satisfies \( \varphi_{init} \)
- `reach(X) :- tr(X,Y), reach(Y).`  \( \mid X \) satisfies \( \varphi_{init} \)
- `reach(X) :- errorConf(X).`  \( \mid X \) satisfies \( \varphi_{error} \)
- + clauses for \( tr \) (the semantics of the programming language)

A program \( prog \) is unsafe w.r.t. \( \varphi_{init} \) and \( \varphi_{error} \) if from an initial configuration satisfying \( \varphi_{init} \) it is possible to reach a final configuration satisfying \( \varphi_{error} \). Otherwise, program \( prog \) is safe.

**Theorem**

\( prog \) is safe iff \( unsafe \notin M(Int) \) (the least model of \( Int \))
Encoding the Verification Problem into CLP

Given the program \textit{increment} and the specification \( \varphi \)

\[
\text{while}(\ast) \{
    x = x + y; \\
    y = y + 1;
\}
\]

\[
\{ x = 1 \land y = 0 \} \text{ increment } \{ x \geq y \}
\]

CLP encoding of program \textit{increment}

A set of \texttt{at(label, command)} facts. while commands are replaced by \texttt{ite} and \texttt{goto}.

\[
\text{at}(l_0, \texttt{ite}(\text{nondet}, l_1, l_h)). \\
\text{at}(l_1, \texttt{asgn}(x, \texttt{plus}(x, y))). \\
\text{at}(l_2, \texttt{asgn}(y, \texttt{plus}(y, 1))). \\
\text{at}(l_3, \texttt{goto}(l_0)). \\
\text{at}(l_h, \texttt{halt}).
\]

CLP encoding of \( \varphi_{\text{init}} \) and \( \varphi_{\text{error}} \)

\[
\text{initConf}(l_0, X, Y) :\neg X = 1, Y = 0. \\
\text{errorConf}(l_h, X, Y):\neg X < Y.
\]
Transforming CLP Programs

Rule-based program transformation

- transformation rules:
  
  \[ R \in \{ \text{Unfolding, Clause Removal, Definition, Folding} \} \]

  \[ \text{Theorem} \]
  
  \[ \text{unsafe} \in M(P) \iff \text{unsafe} \in M(\text{TransfP}) \]

- the transformation rules preserve the least model:

- the rules must be guided by a strategy.
The Unfold/Fold Transformation Strategy

Transform($P$)

\[ \text{TransfP} = \emptyset; \]
\[ \text{Defs} = \{\text{unsafe} :- \text{initConf}(X), \text{reach}(X)\}; \]
\[ \text{while } \exists q \in \text{Defs} \text{ do} \]
\[ \quad \% \text{execute a symbolic evaluation step (resolution)} \]
\[ \quad \text{Cls} = \text{Unfold}(q); \]
\[ \quad \% \text{remove unsatisfiable and subsumed clauses} \]
\[ \quad \text{Cls} = \text{ClauseRemoval}(\text{Cls}); \]
\[ \quad \% \text{introduce new predicates (e.g., a loop invariant)} \]
\[ \quad \text{Defs} = (\text{Defs} - \{q\}) \cup \text{Define}(\text{Cls}); \]
\[ \quad \% \text{match a predicate definition} \]
\[ \quad \text{TransfP} = \text{TransfP} \cup \text{Fold}(\text{Cls}, \text{Defs}); \]
\[ \text{od} \]
The specialization of \textit{Int} w.r.t. \textit{prog} removes all references to:

- \texttt{tr} (i.e., the operational semantics of the imperative language)
  \[
  L: \text{goto } L1;
  \text{tr(cf(cmd(L/goto(L1)),S), cf(C,S)) :- at(L1,C)}.
  \]

- \texttt{at} (i.e., the encoding of \textit{prog})

The Specialized Interpreter for \textit{increment} (Verification Conditions)

\begin{verbatim}
unsafe :- X=1, Y=0, new1(X,Y).
new1(X,Y) :- X=X+Y, Y=Y+1, new1(X,Y).
new1(X,Y) :- X<Y.
\end{verbatim}

New predicates correspond to a subset of the \textit{program points}:

\[
\text{new1(X,Y) :- reach(cf(cmd(0,ite(...)), [
  \text{[[int(x),X],[int(y),Y]]}]))}.
\]
Satisfiability of a set of clauses can be reduced to the standard top-down query evaluation.

The recursive predicate \texttt{new1}, generates an infinite derivation for \texttt{unsafe}.

Top-down evaluation with \textit{tabling} (i.e. memoing of partial answers) does not terminate.
Failure Tabled CLP (FTCLP)

Interpolating Horn Clause (IHC) Solver:

- interpolation provides learned facts from failure that can be used for pruning search.

Failure Tabled CLP (FTCLP): [Navas et al.]

- an interpolating Horn Clause (IHC) Solver
- execute a set of CLP clauses top-down while labelling nodes in the derivation tree with interpolants:

  1. Whenever a loop is detected its execution stops, and it backtracks to an ancestor choice point,
  2. After completion of a subtree, the tabling mechanism will attempt at proving that the predicate where the execution was frozen can be subsumed by any of its ancestors using an interpolant as the subsumption condition
  3. If it fails then its execution is re-activated and the process continues.
Proving Satisfiability of VCs

1. unsafe :- X=1, Y=0, new1(X,Y).
2. new1(X,Y) :- X=X+Y, Y=Y+1, new1(X,Y).
3. new1(X,Y) :- X<Y.

(a) freeze the execution of the recursive clause 2

(b) learn $X \geq Y$ from the failed derivation: $X=1$, $Y=0$, $X<Y$ (compute an interpolant between $X=1$, $Y=0$ and $X<Y$)

(c) check if $X \geq Y$ is an inductive invariant

Unfortunately, $X \geq Y$, $X1=X+Y$, $Y1=Y+1 \not\models X1 \geq Y1$

$X \geq Y$ is not an inductive invariant
Transforming Verification Conditions

Program transformation:
- propagates constraints,
- introduces predicate definitions (i.e., program invariants)

Use of generalization operators:
- to ensure the termination of the transformation,
- to generate program invariants,

... two somewhat conflicting requirements:
- efficiency, to introduce as few definitions as possible,
- precision, to prove as many properties as possible.

Generalization operators add new constraints to predicate definitions that might make the top-down (or bottom-up) evaluation terminating.
Definitions are arranged as a tree:

```
unsafe :- i, A

... newp :- c, B

newq :- d, B

newr :- g, B
```

Generalization operators based on **widening** and **convex-hull**.
The verification conditions are specialized w.r.t. $\varphi_{init}$.

**Specialized Verification Conditions for increment**

... **propagating** the constraint $X=1, Y=0$.

- **unsafe** :- X=1, Y=0, new4(X,Y).
- new4(X,Y) : X=1, Y=0, X1=1, Y1=1, new5(X1,Y1).
- new5(X,Y) : X=1, Y$\geq$0, new8(X,Y).
- new8(X,Y) :- X=1, X1=Y+1, X1$\geq$1, Y1=X1, new9(X1,Y1).
- new8(X,Y) :- X=1, Y$\geq$0, new10(X,Y).
- new10(X,Y) :- X=1, Y$\geq$2.
- new9(X,Y) :- X$\geq$1, Y$\geq$0, new13(X,Y).
- new13(X,Y) :- X1=X+Y, Y1=Y+1, new9(X1,Y1).
- new13(X,Y) :- X$\geq$1, Y$\geq$0, new15(X,Y).
- new15(X,Y) :- X$\geq$1, X$\leq$Y-1.

The transformation adds new constraint $X$\(\geq\)1, $Y$\(\geq\)0, so that FTCLP solver terminates.
Analysing the Specialized VCs

Execution of Recursive CHCs

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X = 2, Y = 2

new9(X, Y)

X ≥ 1, Y ≥ 0

new13(X, Y)

X1 = X + Y, Y1 = Y + 1

new9(X1, Y1)

X ≥ 1, Y ≥ 0

new15(X, Y)

X ≥ 1, X ≤ Y − 1

X = 2, Y = 2 ⊨ X ≥ Y (by definition of interpolation)

X ≥ Y, X ≥ 1, Y ≥ 0, X1 = X + Y, Y1 = Y + 1 ⊨ X1 ≥ Y1
Discovering program invariants

Program transformation and FTCLP improve on infinite failure.

- generalization operators may discover invariants by looking at the history of the computation
e.g., from $X=1$, $Y=0$ and $X=2$, $Y=1$ (one loop execution) by generalization we derive $X\geq 1$, $Y\geq 0$

- interpolation discovers invariants by looking at failed executions
e.g., from $X=1$, $Y=0$ and $X<Y$ we derive $X\geq Y$.

If the invariants are not strong enough to prove the correctness of the program, we iterate the transformation process.
Program Reversal

By specializing

\[
\begin{align*}
\text{unsafe} & :\neg \textit{initial}(A), \text{reach}(A). \\
\text{reach}(A) & :\neg \text{tr}(A,B), \text{reach}(B). \\
\text{reach}(X) & :\neg \text{error}(A).
\end{align*}
\]

w.r.t. unsafe, we propagate the constraint of the initial configuration $\varphi_{\text{init}}$.

By specializing

\[
\begin{align*}
\text{unsafe} & :\neg \textit{error}(A), \text{reach}(A). \\
\text{reach}(B) & :\neg \text{tr}(A,B), \text{reach}(A). \\
\text{reach}(X) & :\neg \textit{initial}(A).
\end{align*}
\]

w.r.t. unsafe, we propagate the constraint of the error configuration $\varphi_{\text{error}}$.

\[
\text{unsafe} \in M(\text{Int}) \text{ iff } \text{unsafe} \in M(\text{RevInt})
\]
A Tool for Verifying Programs through Transformations

- **CIL (C Intermediate Language)** by Necula et al.
- **MAP Transformation System** by the MAP group

Available at: http://map.uniroma2.it/VeriMAP
The architecture of the VeriMAP tool with FTCLP.

The FTCLP solver is implemented using:

- Ciao prolog system
- MathSAT (for the interpolants generation)

Available at: http://code.google.com/p/ftclp
## Experimental evaluation

<table>
<thead>
<tr>
<th></th>
<th>FTCLP</th>
<th>VeriMAP\textsubscript{$M$}</th>
<th>VeriMAP\textsubscript{$M$} + FTCLP</th>
<th>VeriMAP\textsubscript{$PH$}</th>
<th>VeriMAP\textsubscript{$PH$} + FTCLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>answers</td>
<td>116</td>
<td>128</td>
<td>160</td>
<td>178</td>
<td>182</td>
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<tr>
<td>crashes</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
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<tr>
<td>timeouts</td>
<td>95</td>
<td>88</td>
<td>54</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>total time</td>
<td>12470.26</td>
<td>11285.77</td>
<td>9714.41</td>
<td>5678.09</td>
<td>6537.17</td>
</tr>
<tr>
<td>average time</td>
<td>107.50</td>
<td>88.17</td>
<td>60.72</td>
<td>31.90</td>
<td>35.92</td>
</tr>
</tbody>
</table>

Table: Verification results using VeriMAP, FTCLP, and the combination of VeriMAP and FTCLP. The timeout limit is two minutes. Times are in seconds.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>VeriMAP\textsubscript{$M$}</th>
<th>VeriMAP\textsubscript{$M$} + FTCLP</th>
<th>VeriMAP\textsubscript{$PH$}</th>
<th>VeriMAP\textsubscript{$PH$} + FTCLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74</td>
<td>119</td>
<td>104</td>
<td>136</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>38</td>
<td>54</td>
<td>34</td>
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<td>3</td>
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<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Table: Number of definite answers computed by VeriMAP and by the combination of VeriMAP and FTCLP within the first five iterations.
Conclusions and Future Work

- Parametric verification framework (semantics and logic)
  - CLP as a metalanguage
  - Semantics preserving transformations to:
    - iterate specialization and analysis, and
    - pass information between verifiers
  thereby resulting in an incremental verification process.
- We instantiated the verification framework by integrating:
  - an Iterated Specialization tool (VeriMAP), and
  - an Interpolating Horn Clauses Solver (FTCLP).
- Future work: combine these tools in a more synergistic way
  - leverage the partial information FTCLP discovers and integrate it into the Specialized program,
  - refining the generalization step by using the interpolants computed by FTCLP, and
  - use interpolation during the transformation process.