

Synthesizing Concurrent Programs using Answer Set Programming

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Related work

Automated Synthesis of Concurrent Programs

- ▶ E. M. Clarke and E. A. Emerson
[Design and Synthesis of Synchronization Skeletons Using Branching Temporal Logic](#)
Workshop on Logic of Program, Springer-Verlag, 1982
- ▶ Z. Manna and P. Wolper
[Synthesis of Communicating Process from Temporal Specifications](#)
ACM TOPLAS, 1984
- ▶ ...

Answer Set Programming

- ▶ S. Heymans, D. Van Nieuwenborgh and D. Vermeir
[Synthesis from Temporal Specifications using Preferred Answer Set Programming](#)
LNCS no. 3701, 2005
- ▶ ...

Overview

- ▶ Concurrent Programs
 - * Syntax
 - * Semantics
- ▶ Specification of Concurrent Programs
 - * Behavioural properties
 - * Structural properties
 - * Symmetric Concurrent Programs
- ▶ Automated Synthesis of Concurrent Program
based on Answer Set Programming
- ▶ Experimental Results

k -Process Concurrent Program

Syntax

- * P_1, \dots, P_k , *processes*
- * s_1, \dots, s_k , *local variables* ranging over a finite domain L
- * x , a single *shared variable* ranging over a finite domain D

$s_1 := l_1; \dots; s_k := l_k; x := d_0; \underline{\text{do}} P_1 \parallel \dots \parallel P_i \parallel \dots \parallel P_k \underline{\text{od}}$

$\text{true} \rightarrow \underline{\text{if}} \text{gc}_1 \parallel \dots \parallel \text{gc}_h \parallel \dots \parallel \text{gc}_n \underline{\text{fi}}$

$s_i = l \wedge x = d \rightarrow s_i := l'; x := d';$

where $l_1, \dots, l_k, l, l' \in L$, and $d_0, d, d' \in D$.

Example

Syntax

A 2-Process Concurrent Program C

- ▶ P_1, P_2 , processes
- ▶ s_1, s_2 , local variables ranging over $L = \{t, u\}$
- ▶ x , a single shared variable ranging over $D = \{0, 1\}$

$C: \quad s_1 := t; s_2 := t; x := 0; \underline{\text{do}} P_1 \parallel P_2 \underline{\text{od}}$

t: non critical section;
u: critical section;

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u: critical section;

Example

Syntax

A 2-Process Concurrent Program C

- ▶ P_1, P_2 , processes
- ▶ s_1, s_2 , local variables ranging over $L = \{t, u\}$
- ▶ x , a single shared variable ranging over $D = \{0, 1\}$

$C: \quad s_1 := t; s_2 := t; x := 0; \underline{\text{do}} P_1 \parallel P_2 \underline{\text{od}}$

$P_1: \text{true} \rightarrow \underline{\text{if}}$

$s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0;$

$\parallel s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1;$

$\underline{\text{fi}}$

$P_2: \text{true} \rightarrow \underline{\text{if}}$

$s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1;$

$\parallel s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0;$

$\underline{\text{fi}}$

k -Process Concurrent Program

Semantics

A k -Process Concurrent Program C

- * P_1, \dots, P_k , processes
- * s_1, \dots, s_k , local variables ranging over a finite domain L
- * x , a single *shared variable* ranging over a finite domain D

$s_1 := \ell_1; \dots; s_k := \ell_k; x := d_0; \underline{\text{do}} P_1 \parallel \dots \parallel P_i \parallel \dots \parallel P_k \underline{\text{od}}$

The **Kripke structure** $\mathcal{K} = \langle S, S_0, R, \lambda \rangle$ associated with C :

- * set of *states*: $S = L^k \times D$
- * set of *initial states*: $S_0 = \{ \langle \ell_1, \dots, \ell_k, d_0 \rangle \}$
- * total *transition relation*: $R \subseteq S \times S$
- * total *labelling function*: $\lambda : S \rightarrow \mathcal{P}(\text{Elem})$

Example

$$\begin{array}{l} s_1 := t; s_2 := t; x := 0; \\ \underline{\text{do}} \quad \text{true} \rightarrow \underline{\text{if}} \\ \quad s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0; \\ \quad \boxed{ s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1; } \\ \quad \underline{\text{fi}} \end{array} \left\| \begin{array}{l} \text{true} \rightarrow \underline{\text{if}} \\ \quad s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1; \\ \quad \boxed{ s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0; } \\ \quad \underline{\text{fi}} \end{array} \quad \underline{\text{od}}$$

Example (Cont'd)

$s_1 := t; s_2 := t; x := 0;$

$\underline{\text{do}}$	$\text{true} \rightarrow \underline{\text{if}}$	$\left\ \right.$	$\text{true} \rightarrow \underline{\text{if}}$	$\underline{\text{od}}$
	$s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0;$		$s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1;$	
	$\left[s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1; \right.$		$\left. \left[s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0; \right. \right.$	
	$\left. \underline{\text{fi}} \right.$		$\left. \underline{\text{fi}} \right.$	

$S_0 = \{\langle t, t, 0 \rangle\}$

$\langle t, t, 0 \rangle$

$S = \{\langle t, t, 0 \rangle, \dots\}$

$\lambda = \{\langle \langle t, t, 0 \rangle, \{s_1 = t, s_2 = t, x = 0\} \rangle, \dots\}$

Example (Cont'd)

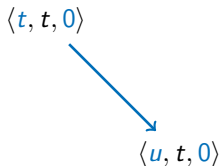
$s_1 := t; s_2 := t; x := 0;$

$\underline{\text{do}}$	$\underline{\text{true}} \rightarrow \underline{\text{if}}$	$\underline{\text{true}} \rightarrow \underline{\text{if}}$	$\underline{\text{od}}$
	$\frac{s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0;}{\boxed{s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1;}}$	$\frac{s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1;}{\boxed{s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0;}}$	
	$\underline{\text{fi}}$	$\underline{\text{fi}}$	

$S_0 = \{\langle t, t, 0 \rangle\}$

$S = \{\langle t, t, 0 \rangle, \langle u, t, 0 \rangle, \dots\}$

$R = \{\langle \langle t, t, 0 \rangle, \langle u, t, 0 \rangle \rangle, \dots\}$



$\lambda = \{\langle \langle t, t, 0 \rangle, \{s_1 = t, s_2 = t, x = 0\} \rangle, \dots\}$

Example (Cont'd)

$s_1 := t; s_2 := t; x := 0;$

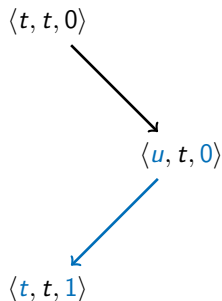
$\text{do } \frac{\text{true} \rightarrow \underline{\text{if}} \quad \begin{array}{l} s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0; \\ \boxed{\frac{s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1;}{\underline{\text{fi}}}} \end{array}}{\underline{\text{fi}}}$		$\text{true} \rightarrow \underline{\text{if}} \quad \begin{array}{l} s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1; \\ \boxed{\frac{s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0;}{\underline{\text{fi}}}} \end{array} \quad \text{od}$
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$S_0 = \{\langle t, t, 0 \rangle\}$

$S = \{\langle t, t, 0 \rangle, \langle u, t, 0 \rangle, \langle t, t, 1 \rangle, \dots\}$

$R = \{\langle \langle t, t, 0 \rangle, \langle u, t, 0 \rangle \rangle, \langle \langle u, t, 0 \rangle, \langle t, t, 1 \rangle \rangle, \dots\}$

$\lambda = \{\langle \langle t, t, 0 \rangle, \{s_1 = t, s_2 = t, x = 0\} \rangle, \dots\}$



Example (Cont'd)

$s_1 := t; s_2 := t; x := 0;$

$\underline{\text{do}}$ $\text{true} \rightarrow \underline{\text{if}}$
 $s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0;$
 $\parallel s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1;$
 $\underline{\text{fi}}$

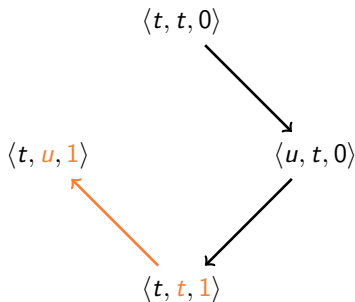
$\text{true} \rightarrow \underline{\text{if}}$
 $s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1;$ $\underline{\text{od}}$
 $\parallel s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0;$
 $\underline{\text{fi}}$

$S_0 = \{\langle t, t, 0 \rangle\}$

$S = \{\langle t, t, 0 \rangle, \langle u, t, 0 \rangle,$
 $\langle t, t, 1 \rangle, \langle t, u, 1 \rangle, \dots\}$

$R = \{\langle \langle t, t, 0 \rangle, \langle u, t, 0 \rangle \rangle,$
 $\langle \langle u, t, 0 \rangle, \langle t, t, 1 \rangle \rangle,$
 $\langle \langle t, t, 1 \rangle, \langle t, u, 1 \rangle \rangle, \dots\}$

$\lambda = \{\langle \langle t, t, 0 \rangle, \{s_1 = t, s_2 = t, x = 0\} \rangle, \dots\}$



Example (Cont'd)

$s_1 := t; s_2 := t; x := 0;$

$\underline{\text{do}}$ $\text{true} \rightarrow \underline{\text{if}}$
 $s_1 = t \wedge x = 0 \rightarrow s_1 := u; x := 0;$
 $\parallel s_1 = u \wedge x = 0 \rightarrow s_1 := t; x := 1;$
 $\underline{\text{fi}}$

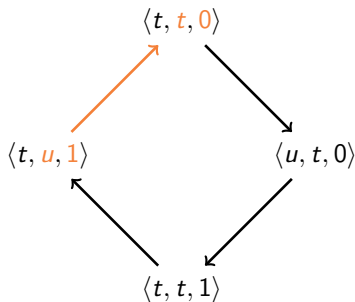
$\text{true} \rightarrow \underline{\text{if}}$
 $s_2 = t \wedge x = 1 \rightarrow s_2 := u; x := 1;$ $\underline{\text{od}}$
 $\parallel s_2 = u \wedge x = 1 \rightarrow s_2 := t; x := 0;$
 $\underline{\text{fi}}$

$S_0 = \{\langle t, t, 0 \rangle\}$

$S = \{\langle t, t, 0 \rangle, \langle u, t, 0 \rangle,$
 $\langle t, t, 1 \rangle, \langle t, u, 1 \rangle, \dots\}$

$R = \{\langle \langle t, t, 0 \rangle, \langle u, t, 0 \rangle \rangle,$
 $\langle \langle u, t, 0 \rangle, \langle t, t, 1 \rangle \rangle,$
 $\langle \langle t, t, 1 \rangle, \langle t, u, 1 \rangle \rangle,$
 $\langle \langle t, u, 1 \rangle, \langle t, t, 0 \rangle \rangle\}$

$\lambda = \{\langle \langle t, t, 0 \rangle, \{s_1 = t, s_2 = t, x = 0\} \rangle, \dots\}$



Specification: Behavioural Properties

Time dependant behavioural properties of Concurrent Programs:

- ▶ safety
- ▶ liveness

Specified in a Temporal Logic, i.e., Computation Tree Logic (CTL):

- ▶ path quantifiers: for all paths **A**, for some paths **E**
- ▶ temporal operators: eventually **F**, globally **G**, next **X**,....

A k -Process Concurrent Program C satisfies a behavioural property φ iff the Kripke structure \mathcal{K} associated with C satisfies φ .

Specification: Behavioural Properties

Computation Tree Logic

- ▶ Syntax: $\varphi ::= b \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \text{EX}\varphi \mid \text{EG}\varphi \mid \text{E}[\varphi_1 \text{U}\varphi_2]$
 $\text{AG}\varphi \equiv \neg\text{EF}\neg\varphi$, etc.

- ▶ Semantics:
$$\left. \begin{array}{l} \text{Kripke structure } \mathcal{K} \\ \text{state } s \\ \text{formula } \varphi \end{array} \right\} \mathcal{K}, s \models \varphi$$

recursively defined as follows:

- $\mathcal{K}, s \models b$ iff $b \in \lambda(s)$
- $\mathcal{K}, s \models \neg\varphi$ iff $\mathcal{K}, s \models \varphi$ does not hold
- $\mathcal{K}, s \models \varphi_1 \wedge \varphi_2$ iff $\mathcal{K}, s \models \varphi_1$ and $\mathcal{K}, s \models \varphi_2$
- $\mathcal{K}, s \models \text{EX}\varphi$ iff there exists $\langle s, t \rangle \in R$ such that $\mathcal{K}, t \models \varphi$
- $\mathcal{K}, s \models \text{EG}\varphi$ iff there exists a path π such that $\pi_0 = s$ and for all $i \geq 0$, $\mathcal{K}, \pi_i \models \varphi$
- $\mathcal{K}, s \models \text{E}[\varphi_1 \text{U}\varphi_2]$ iff there exists a path $\pi = \langle s, s_1, \dots \rangle$ in \mathcal{K} and $i \geq 0$ such that $\mathcal{K}, \pi_i \models \varphi_2$ and for all $0 \leq j < i$, $\mathcal{K}, \pi_j \models \varphi_1$

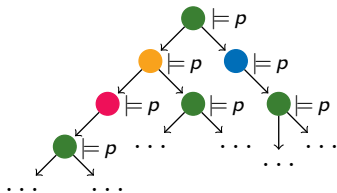
Specification: Behavioural Properties

Computation Tree Logic

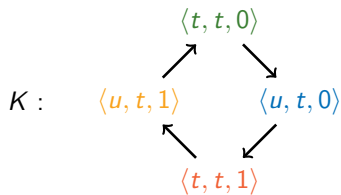
k -Process Concurrent Program
satisfying p



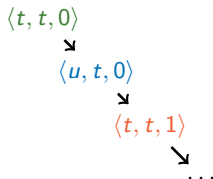
$$M, \bullet \models \text{AG } p$$



2-Process Concurrent Program
satisfying mutual exclusion



$$K, \langle t, t, 0 \rangle \models \text{AG } \neg (s_1 = u \wedge s_2 = u)$$



Specification: Structural Properties

Symmetric Program Structure

- ▶ local transition relation $T \subseteq L \times L$
pattern of execution of each process P_i
- ▶ k -generating function $f: D \rightarrow D$ (permutation on the domain of x)
global property of a concurrent program C

T and f define a **Symmetric Program Structure** $\Sigma = \langle f, T \rangle$

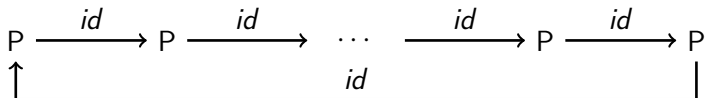
A k -Process Concurrent Program C satisfies Σ iff

1. each process P_1, \dots, P_k in C satisfies T , and
2. C satisfies f

Symmetric Concurrent Programs

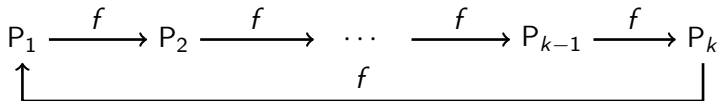
k -generating function f is

either the identity function id



(Dijkstra's semaphore)

or a generator of a cyclic group $\{id, f, \dots, f^{k-1}\}$ of order k



(Peterson's algorithm)

Example

A 2-Process **Symmetric** Concurrent Program for Mutual Exclusion

Symmetric program structure Σ : $T : t \overset{\curvearrowright}{\rightleftarrows} u \quad f : \begin{array}{c} 0 \\ 1 \end{array} \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} 0 \\ 1 \end{array}$

$true \rightarrow \underline{if} \quad s_1 = t \wedge x = 0 \rightarrow s_1 := u ; x := 0 ;$

$\square \quad s_1 = t \wedge x = 1 \rightarrow s_1 := t ; x := 1 ;$

$\square \quad s_1 = u \wedge x = 0 \rightarrow s_1 := t ; x := 1 ; \underline{fi}$

Example

A 2-Process **Symmetric** Concurrent Program for Mutual Exclusion

Symmetric program structure Σ : $T : t \overset{\curvearrowright}{\rightleftarrows} u \quad f : \begin{matrix} 0 & \rightarrow & 0 \\ 1 & \times & 1 \end{matrix}$

$true \rightarrow \underline{if} \quad s_1 = t \wedge x = 0 \rightarrow s_1 := u ; x := 0 ;$
 $true \rightarrow \underline{if} \quad s_2 = t \wedge x = 1 \rightarrow s_2 := u ; x := 1 ;$

$\boxed{ s_1 = t \wedge x = 1 \rightarrow s_1 := t ; x := 1 ;$
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$\boxed{ s_1 = u \wedge x = 0 \rightarrow s_1 := t ; x := 1 ; \underline{fi}$
 $\boxed{ s_2 = u \wedge x = 1 \rightarrow s_2 := t ; x := 0 ; \underline{fi}$

Synthesis of a Concurrent Programs

- ▶ automatically derive a k -concurrent program from a given specification:
 - ▶ Behavioural Properties
 - ▶ Structural Properties
- ▶ derive a program correct by construction

$s_1 := l_1; s_2 := l_2; x := d_0;$

$$\text{do} \left[\begin{array}{l} \text{true} \rightarrow \underline{\text{if}} \\ \quad s_1 = ? \wedge x = ? \rightarrow s_1 := ?; x := ?; \\ \quad \dots \\ \quad s_1 = ? \wedge x = ? \rightarrow s_1 := ?; x := ?; \\ \quad \underline{\text{fi}} \end{array} \right] \parallel \left[\begin{array}{l} \text{true} \rightarrow \underline{\text{if}} \\ \quad s_2 = ? \wedge x = ? \rightarrow s_2 := ?; x := ?; \\ \quad \dots \\ \quad s_2 = ? \wedge x = ? \rightarrow s_2 := ?; x := ?; \\ \quad \underline{\text{fi}} \end{array} \right] \text{od}$$

Synthesis Problem

Given:

1. a CTL formula φ (Behavioural Property),
2. a Symmetric Program Structure $\Sigma = \langle f, T \rangle$ (Structural Property)

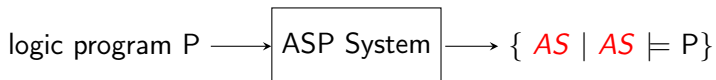
we look for a k -process concurrent program C such that

- * for all $i > 0$ $f(P_i) = P_{(i \bmod k)+1}$, and
- * C satisfies φ

We **guess** P_1 satisfying T and we **generate** the set P_2, \dots, P_k using f such that the Kripke structure associated with C satisfies φ

Answer Set Programming (ASP)

- ▶ knowledge representation and reasoning tool
 - logic **program** \Rightarrow encoding of a **problem**
 - Answer Set** \Rightarrow **solution** of a problem
- ▶ guess & check
 - ▶ guess: rules for generate candidate solutions
 - ▶ check: rules for eliminate invalid candidates



ASP-based Synthesis Procedure

We reduce the derivation of a k -Process Concurrent Program
to an answer set computation

$$\varphi + \Sigma$$

ASP-based Synthesis Procedure

We reduce the derivation of a k -Process Concurrent Program to an answer set computation

Encoding: a logic program Π encodes a *synthesis problem*.

Π is the union of:

- ▶ Π_φ , encoding Behavioural Properties φ
- ▶ Π_Σ , encoding Structural Properties Σ



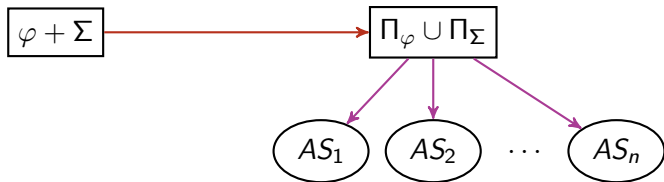
ASP-based Synthesis Procedure

We reduce the derivation of a k -Process Concurrent Program to an **answer set computation**

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ASP-based Synthesis Procedure

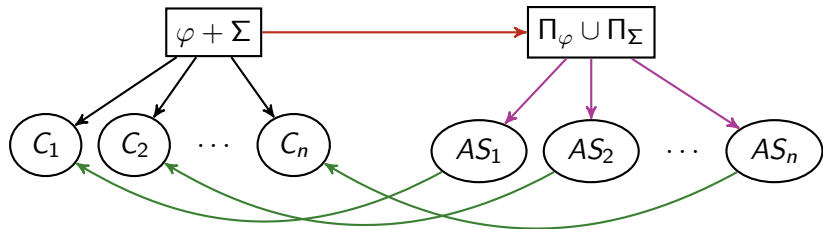
We reduce the **derivation** of a k -Process Concurrent Program to an **answer set computation**

Encoding: a logic program Π encodes a *synthesis problem*.

Π is the union of:

- ▶ Π_φ , encoding Behavioural Properties φ
- ▶ Π_Σ , encoding Structural Properties Σ

Decoding: answer sets $ans(\Pi) = \{AS_1, AS_2, \dots, AS_n\}$ encode *solutions* of the synthesis problem



Example

Encoding

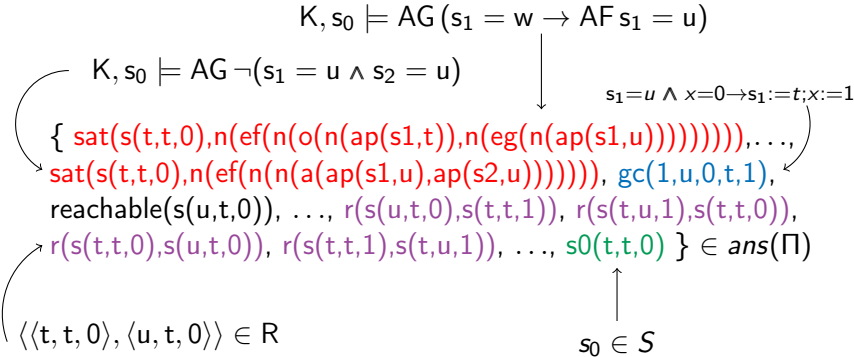
Symmetric program structure Σ : $T : t \overset{\curvearrowright}{\rightleftarrows} u \quad f : \begin{array}{cc} 0 & 0 \\ 1 & 1 \end{array}$

Behavioural property: $\varphi = \text{AG } \neg(s_1 = u \wedge s_2 = u)$

$$\begin{aligned} \Pi = \{ & \\ & \leftarrow \text{not sat}(s(t, t, 0), n(\text{ef}(a(\text{ap}(s_1, u), \text{ap}(s_2, u)))). \\ & \quad \text{sat}(s(t, t, 0), n(\text{ef}(a(\text{ap}(s_1, u), \text{ap}(s_2, u))))) \leftarrow \\ & \quad \quad \text{not sat}(s(t, t, 0), \text{ef}(a(\text{ap}(s_1, u), \text{ap}(s_2, u)))). \\ & \quad \quad \dots \\ & \quad \text{sat}(s(t, t, 0), a(\text{ap}(s_1, u), \text{ap}(s_2, u))) \leftarrow \\ & \quad \quad \text{sat}(s(t, t, 0), \text{ap}(s_1, u)), \text{sat}(s(t, t, 0), \text{ap}(s_2, u)). \\ & \quad \quad \dots \\ & \quad \text{gc}(1, X, 0, Y, 0) \vee \text{gc}(1, X, 0, Y, 1) \vee \dots \leftarrow \text{reachable}(X, _, 0). \\ & \quad \quad \dots \\ & \quad \text{gc}(2, u, f(1), t, f(0)) \leftarrow \text{gc}(1, u, 0, t, 1). \\ & \} \end{aligned}$$

Example

Decoding



Correctness of Synthesis Procedure

Theorem (Correctness of Synthesis)

Let $\Pi = \Pi_\varphi \cup \Pi_\Sigma$. be the logic program obtained from:

1. a CTL formula φ , and
2. a symmetric program structure $\Sigma = \langle f, T \rangle$.

Then,

$$(s_1 := \ell_0; \dots; s_k := \ell_0; x := d_0; \underline{\text{do}} P_1 \parallel \dots \parallel P_k \underline{\text{od}}) \models \varphi$$

iff there exists an answer set AS in $\text{ans}(\Pi)$ such that

$$\forall i \in \{1, \dots, k\}, \forall \ell, \ell' \in L, \forall d, d' \in D,$$

$$(s_i = \ell \wedge x = d \rightarrow s_i := \ell'; x := d') \text{ is in } P_i \text{ iff } AS \models \text{gc}(i, \ell, d, \ell', d')$$

Experimental results

Examples

ME *Mutual Exclusion*: no two processes are in use for all i, j in $\{1, \dots, k\}$, with $i \neq j$,

$$AG \neg (s_i = u \wedge s_j = u)$$

SF *Starvation Freedom*: if a process is waiting then it will enter in use for all i in $\{1, \dots, k\}$,

$$AG (s_i = w \rightarrow AF s_i = u)$$

BO *Bounded Overtaking*: while a process is waiting, any other process can exit from its critical section at most once for all i, j in $\{1, \dots, k\}$,

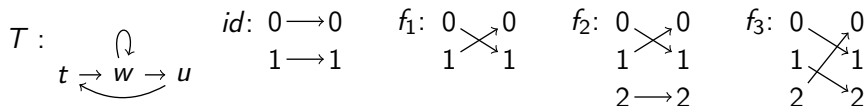
$$AG ((s_i = w \wedge s_j = u) \rightarrow AF (s_j = t \wedge A[\neg(s_j = u) \cup s_i = u]))$$

MR *Maximal Reactivity (MR)*: if a process is waiting and all others are thinking then in the next state it will enter in use for all i in $\{1, \dots, k\}$,

$$AG ((s_i = w \wedge \bigwedge_{j \in \{1, \dots, k\} \setminus \{i\}} s_j = t) \rightarrow EX s_i = u)$$

Experimental results

Synthesis of k -process concurrent programs



Program	Satisfied Properties	$ D $	f	$ ans(\Pi) $	Time (sec)
mutex for 2 processes	ME	2	id	6	0.07
	ME	2	f_1	7	0.70
	ME, SF	2	f_1	3	0.71
	ME, SF, BO	2	f_1	3	1.44
	ME, SF, BO, MR	3	f_2	2	11.70
mutex for 3 processes	ME	2	id	5	0.95
	ME	2	f_1	10	0.87
	ME, SF	3	f_3	8	152.00
	ME, SF, BO	3	f_3	8	1700.00

Conclusions

- ▶ reduction of the **design of a concurrent program** to the design of its **formal specification**
- ▶ fully declarative solution (independent of the ASP solver)
- ▶ state space explosion problem
 - ▶ exploit CTL formulas symmetries,
 - ▶ exploit Kripke structures symmetries,
 - ▶ ...