Verifying relational program properties by transforming constrained Horn clauses

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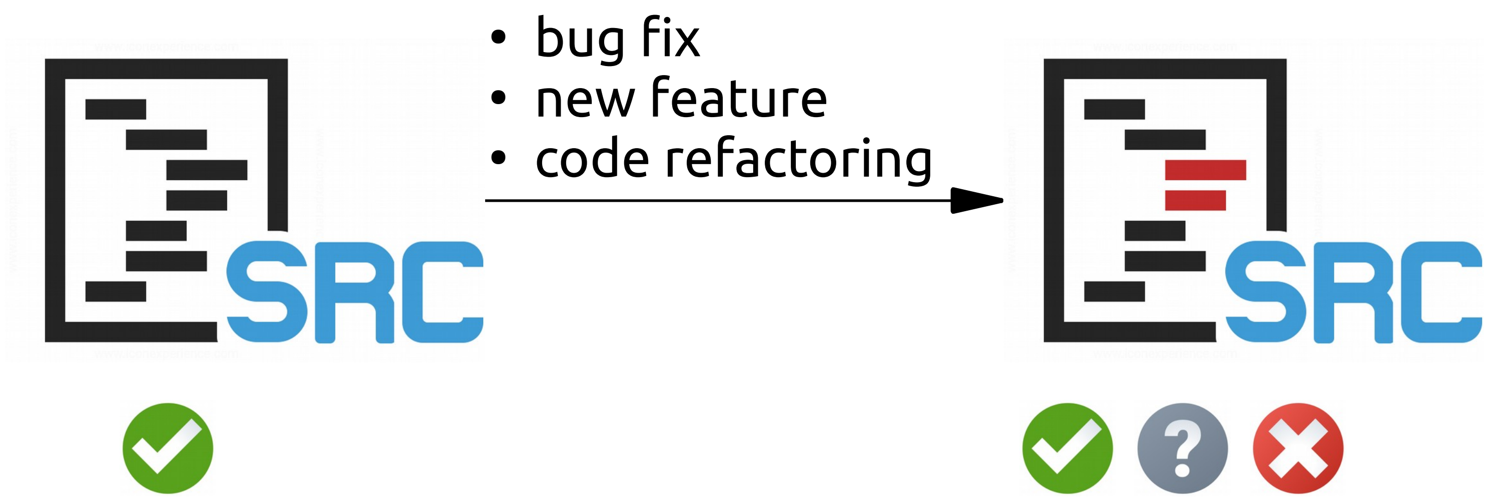
Relational verification



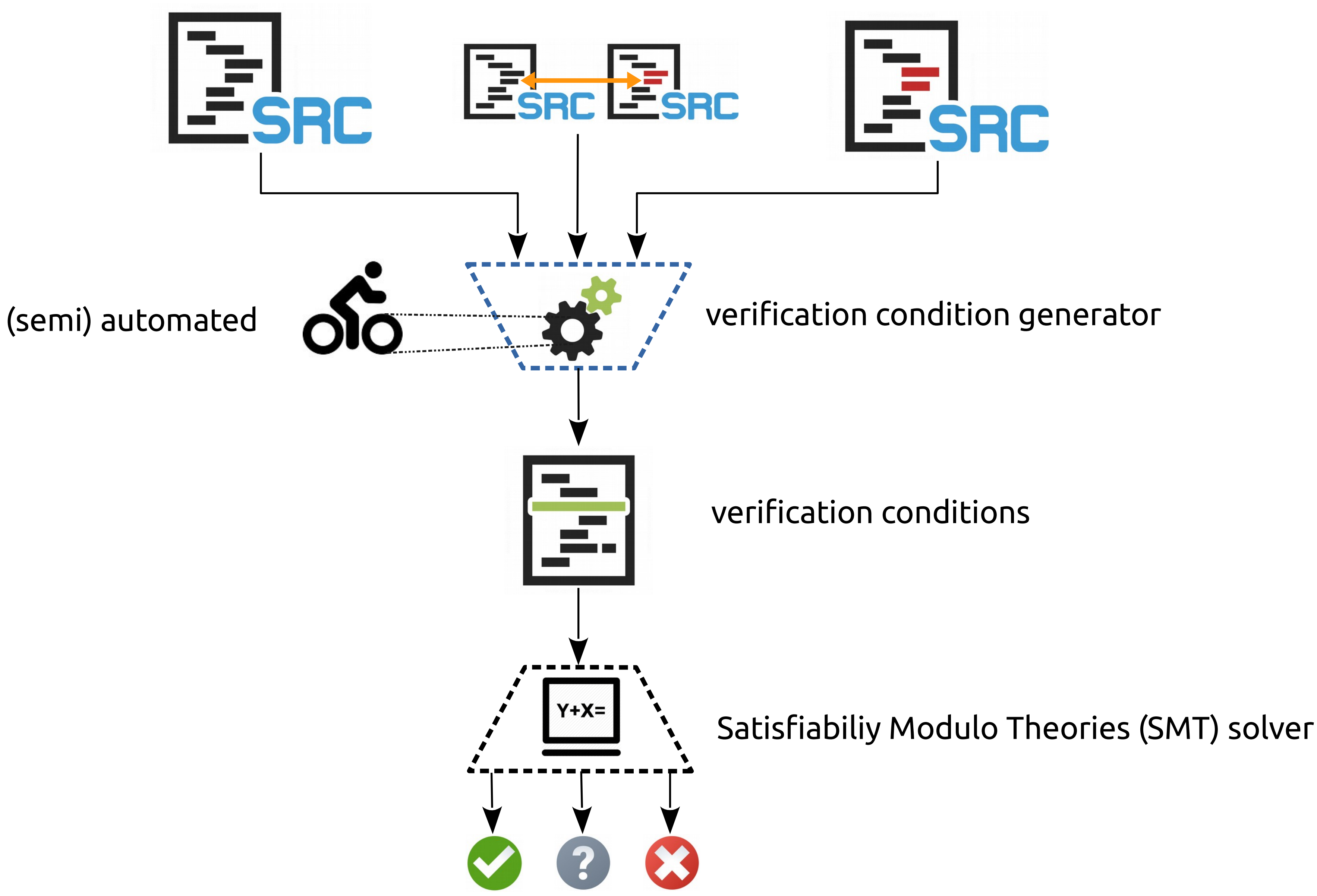
Relational properties

1. modified code still complies with its specification
2. unmodified code has not been affected by the changeset

Example: *program equivalence*

Verification methods

State-of-the-art



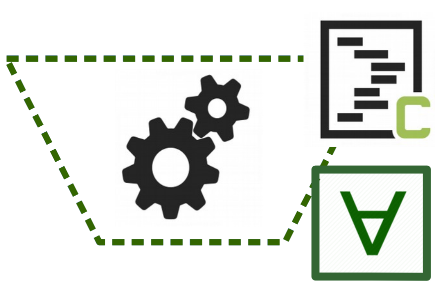
Weakeness

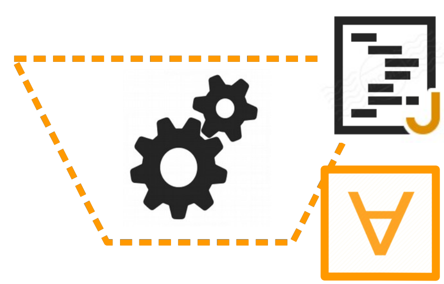
specific for

programming language

properties

verification condition generator





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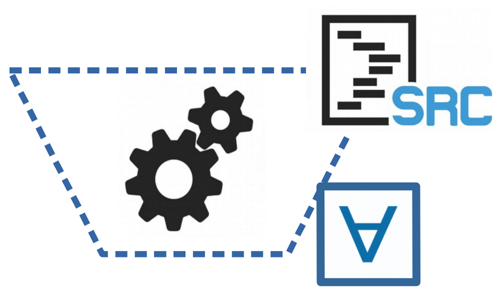
Our goal & contribution

Achieve a higher level of **parametricity** with respect to



programming language &

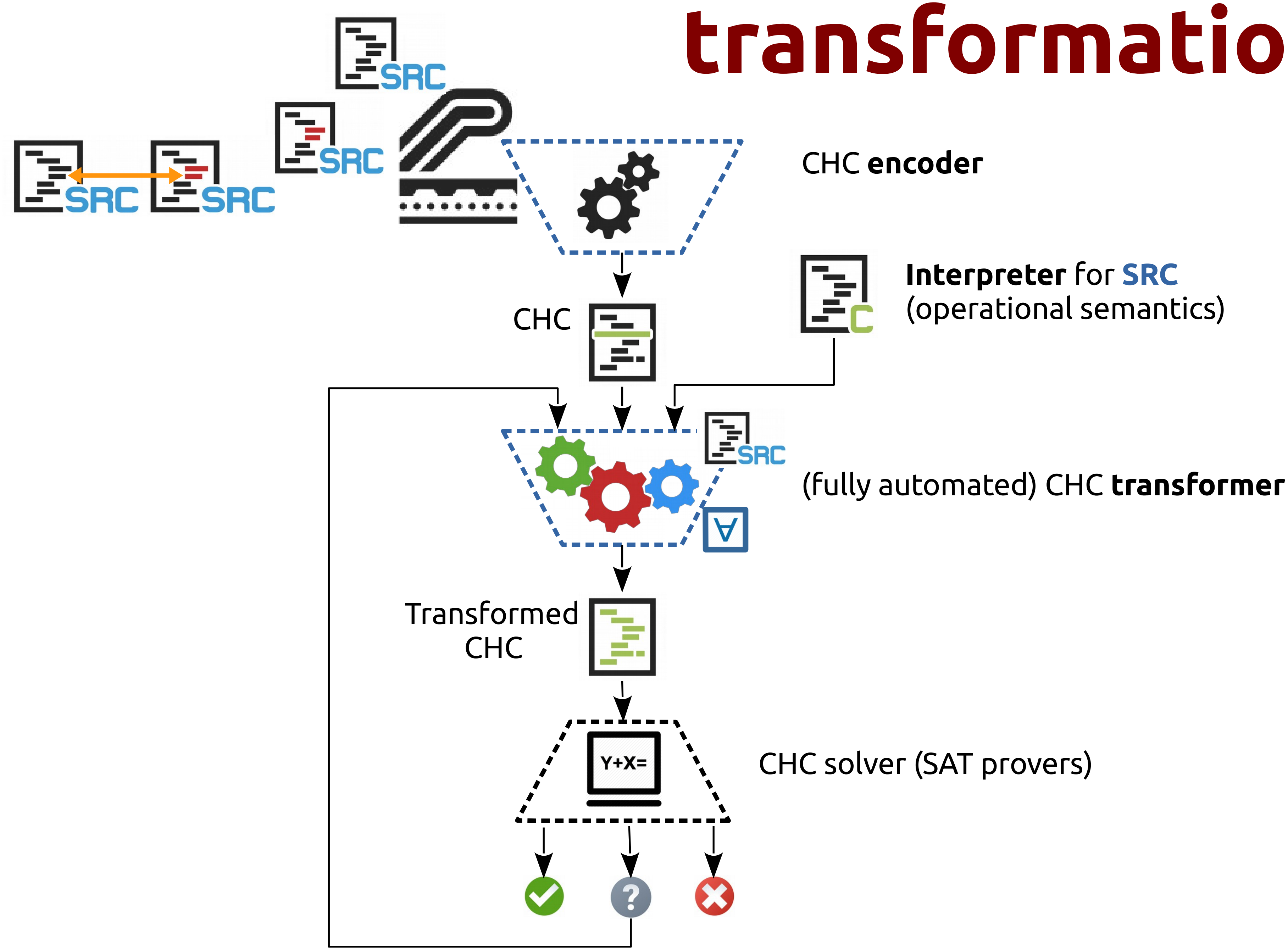
properties

Verification method based on

Transformation of Constrained Horn Clauses (CHCs)

- **CHCs** as a metalanguage for representing  and  as a set of implications of the form $A_0 \leftarrow c, A_1, \ldots, A_n$
- **Transformations** of CHCs to compose clauses representing the programs and the relational property
- **Transformations** increase the effectiveness of satisfiability provers

Relational Verification by CHCs transformation



Specifying Relational Properties using CHCs

The relational property $\{\varphi\}\ P_1 \sim P_2\ \{\psi\}$ is translated into the clause

$$false \leftarrow pre(X,Y), p1(X,X'), p2(Y,Y'), neg\_post(X',Y')$$

pre-relation	φ	$pre(X,Y)$
input/output relation	P_1	$p1(X,X')$
input/output relation	P_2	$p2(Y,Y')$
negation of post-relation	$\neg\psi$	$neg\_post(X',Y')$

check the **validity** of a relational property

reduces to

check the **satisfiability** of its CHCs representation

Interpreter (a glimpse)

Operational semantics of the programming language

$$\underline{prog(X, X')} \leftarrow \underline{initConf(C, X)}, \boxed{reach(C, C')}, \underline{finalConf(C', X')}$$

input/output relation

initial C and final C' configurations

$cf(cmd(Label, Command), Environment)$

$$\boxed{\begin{array}{l} reach(C, C) \\ reach(C, C2) \leftarrow \boxed{tr(C, C1)}, reach(C1, C2) \end{array}}$$

`x=e;`

$$\boxed{\begin{array}{l} tr(cf(cmd(L, asgn(X, expr(E))), Env), cf(cmd(L1, C), Env1)) \leftarrow \\ \quad eval(E, Env, V), update(Env, X, V, Env1), nextlab(L, L1), at(L1, C) \end{array}}$$

Transformation of CHCs

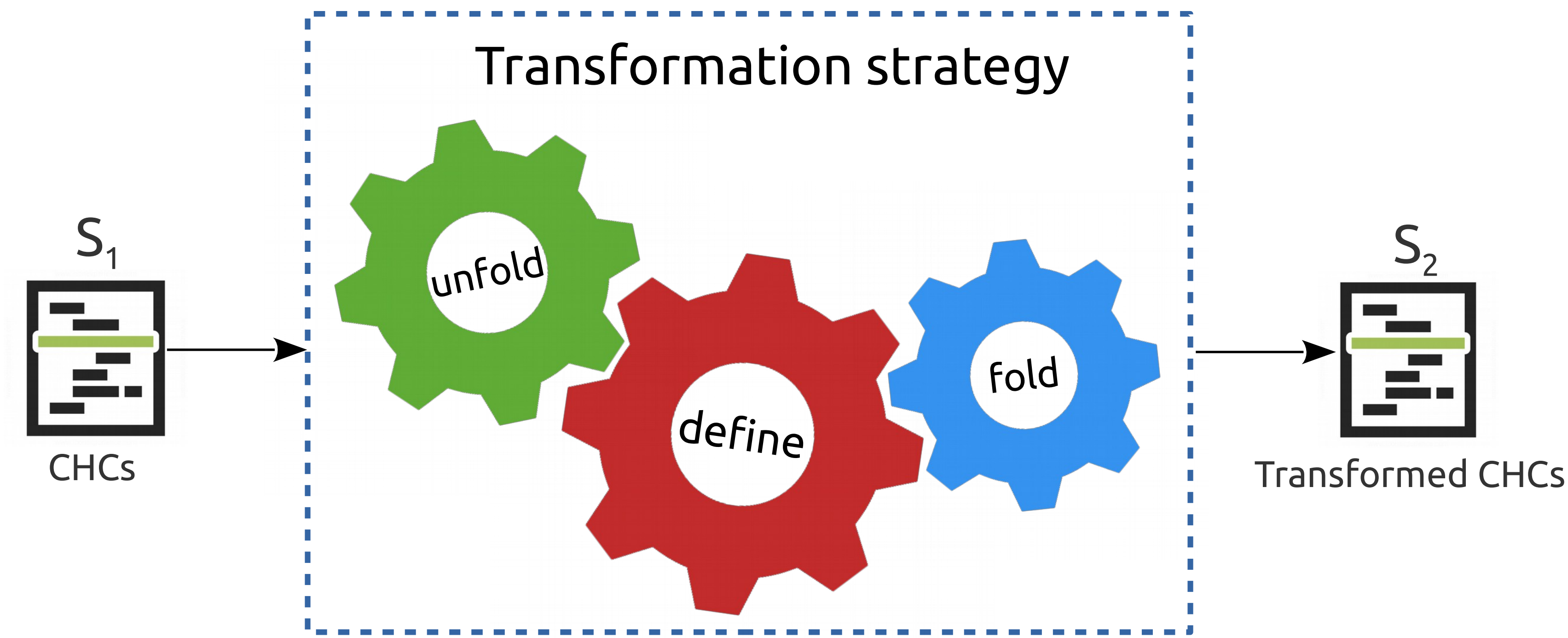
CHS solvers are often unable to prove satisfiability

A technique that

- manipulates clauses
- preserves their satisfiability

Transformation strategies:

1. CHC Specialization
2. Predicate Pairing



S_1 is satisfiable *iff* S_2 is satisfiable

Interpreters & CHC Specialization

Take advantage of static information, that is,

- actual programs
- relational property

to customize the interpreter

By specializing the interpreter w.r.t. the static input, we get CHCs with no references to

- **reach**
- **tr**
- complex terms representing **configurations**

Example

P_1:
```
void sum_upto() {
   z1=f(x1);
}
int f(int n1){
   int r1;
   if (n1 <= 0) {
      r1 = 0;
   } else {
      r1 = f(n1 - 1) + n1;
   }
   return r1;
}
```
non-tail recursive

P_2:
```
void prod() {
   z2 = g(x2,y2);
}
int g(int n2, int m2){
   int r2;
   r2=0;
   while (n2 > 0) {
      r2 += m2;
      n2--;
   }
   return r2;
}
```
iterative

global variables $\mathcal{V}_1$={ x1,z1} of P_1

global variables $\mathcal{V}_2$={ x2,y2,z2}of P_2

$$z1=\sum_{i=0}^{x1} i$$

$$\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$$

$$z2=x2\times y2$$

$$Leq: \ \{\mathtt{x1}=\mathtt{x2},\ \mathtt{x2}\le\mathtt{y2}\}\ \mathtt{sum\_upto} \sim \mathtt{prod}\ \{\mathtt{z1}\le\mathtt{z2}\}$$

CHCs specialization (1)

P_1:
```
void sum_upto() {
  z1=f(x1);
}
int f(int n1){
  int r1;
  if (n1 <= 0) {
    r1 = 0;
  } else {
    r1 = f(n1 - 1) + n1;
  }
  return r1;
}
```

CHCs

$su(X1, Z1') \leftarrow f(X1, Z, X1, R, N1, Z1')$

$f(X, Z, N, R, N, 0) \leftarrow N \leq 0$

$f(X, Z, N, R, N, Z1) \leftarrow N \geq 1, N1 = N - 1, Z1 = R2 + N, f(X, Z, N1, R1, N2, R2)$

CHCs specialization (2)

P_2:
```
void prod() {
   z2 = g(x2,y2);
}
int g(int n2, int m2){
   int r2;
   r2=0;
   while (n2>= 1) {
     r2 += m2;
     n2--;
   }
   return r2;
}
```

CHCs

$$p(X2, Y2, Z2') \leftarrow g(X2, Y2, Z, X2, Y2, 0, N, P, Z2')$$

$$\begin{cases} g(X, Y, Z, N, P, R, N, P, R) \leftarrow N \leq 0 \\ g(X, Y, Z, N, P, R, N2, P2, R2) \leftarrow N \geq 1, N1 = N - 1, R1 = P + R, \\ \qquad g(X, Y, Z, N1, P, R1, N2, P2, R2) \end{cases}$$

Satisfiability of CHCs

$$\{\varphi\}\ P_1\ \sim\ P_2\ \{\psi\}$$

$$false \leftarrow X1 = X2, X2 \leq Y2, Z1' > Z2', su(X1, Z1'), p(X2, Y2, Z2') \qquad (*)$$

$$P_1 \left\{ \begin{array}{l} su(X1, Z1') \leftarrow f(X1, Z, X1, R, N1, Z1') \\ f(X, Z, N, R, N, 0) \leftarrow N \leq 0 \\ f(X, Z, N, R, N, Z1) \leftarrow N \geq 1, N1 = N-1, Z1 = R2+N, f(X, Z, N1, R1, N2, R2) \end{array} \right.$$

$$P_2 \left\{ \begin{array}{l} p(X2, Y2, Z2') \leftarrow g(X2, Y2, Z, X2, Y2, 0, N, P, Z2') \\ g(X, Y, Z, N, P, R, N, P, R) \leftarrow N \leq 0 \\ g(X, Y, Z, N, P, R, N2, P2, R2) \leftarrow N \geq 1, N1 = N-1, R1 = P+R, \\ \qquad\qquad g(X, Y, Z, N1, P, R1, N2, P2, R2) \end{array} \right.$$

- state-of-the-art solvers for CHCs with Linear Integer Arithmetic (LIA) are unable to prove their satisfiability

  to prove their satisfiability, that is, the premise of clause (*) unsatisfiable, LIA solvers should discover quadratic relations

  $$\boxed{Z1' = X1 \times (X1-1)/2} \qquad \boxed{Z2' = X2 \times Y2}$$

- reasoning on su and p separately is unhelpful

Predicate Pairing

- **Solution 1**: use a solver for non-linear integer arithmetic

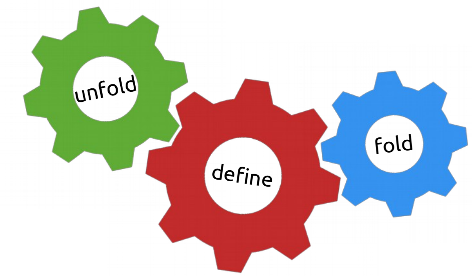
  drawback: satisfiability of constraints is undecidable

- **Solution 2**: use the transformation, again!

  Predicate Pairing transformation strategy

  - composes the predicates f and g into a new predicate fg equivalent to their conjunction
  - Objective: to discover relations among variables occurring in f and g to help the solvers in proving the satisfiability of the CHCs

Predicate Paring in action



$$false \leftarrow X1 = X2, X2 \leq Y2, Z1' > Z2', su(X1, Z1'), p(X2, Y2, Z2')$$

1. Unfold

$$\begin{aligned} false \leftarrow\ & X1 \leq Y2, Z1' > Z2', \\ & f(X1, Z, X1, R, N1, Z1'), g(X1, Y2, Z, X1, Y2, 0, N2, P2, Z2') \end{aligned}$$

2. Define

$$\boxed{\begin{aligned} & fg(X1, Z, N, R, N1, Z1', Y2, V, W, N2, P2, Z2') \leftarrow \\ & \qquad f(X1, Z, N, R, N1, Z1'), g(X1, Y2, V, N, Y2, W, N2, P2, Z2') \end{aligned}}$$

3. Fold

$$false \leftarrow X1 \leq Y2, Z1' > Z2', fg(X1, Z, X1, R, N1, Z1', Y2, Z, 0, N2, P2, Z2')$$

Satisfiability of CHCs

Transformed CHCs

$$
\begin{aligned}
&\mathit{false} \leftarrow X1 \le Y2,\ \ Z1' > Z2',\ \ \mathit{fg}(X1, Z, X1, R, N1, Z1', Y2, Z, 0, N2, P2, Z2') \\
&\mathit{fg}(X, Z, N, R, N, 0, Y2, V, Z2, N, P2, Z2) \leftarrow N \le 0 \\
&\mathit{fg}(X, Z, N, R, N, Z1, Y2, V, W, N2, P2, Z2) \leftarrow \\
&\qquad\qquad N \ge 1,\ \ N1 = N-1,\ \ Z1 = R2+N,\ \ M = Y2+W, \\
&\qquad\qquad \mathit{fg}(X, Z, N1, R1, S, R2, Y2, V, M, N2, P2, Z2)
\end{aligned}
$$

Predicate Pairing makes it possible to infer linear relations among variables in the conjunction fg of predicates f and g

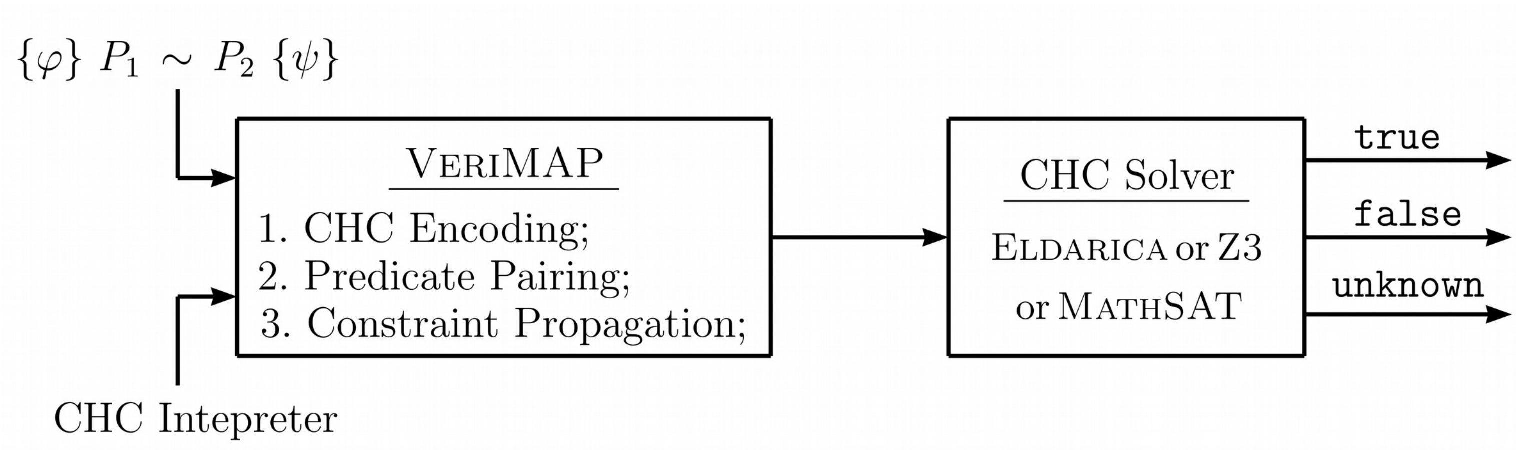
$$
\boxed{
\begin{aligned}
&\mathit{fg}(X1, Z, N, R, N1, Z1', Y2, V, W, N2, P2, Z2') \leftarrow \\
&\qquad f(X1, Z, N, R, N1, Z1'), g(X1, Y2, V, N, Y2, W, N2, P2, Z2')
\end{aligned}
}
$$

The conjunction fg enforces the linear constraint

$$\boxed{(X1 > Y2) \vee (Z1' \le Z2')}$$

Hence the satisfiability of the first clause

Implementation & Experimental Evaluation



Properties	*n*	*Enc+Eld*	*Enc+Z3*	*PP+Eld*	*PP+MS*	*PP+Z3*	*CP+Eld*	*CP+MS*	*CP+Z3*
ITERATIVE	21	7	6	13	19	6	17	20	21
RECURSIVE	18	7	8	7	11	6	14	11	13
ITERATIVE-RECURSIVE	4	0	0	0	3	0	4	4	4
ARRAYS	5	0	1	1	1	4	2	2	5
LEQ	6	1	1	1	6	1	3	6	4
MONOTONICITY	18	4	4	11	16	8	11	17	14
INJECTIVITY	11	0	0	0	11	4	7	11	10
FUNCTIONALITY	7	5	5	6	7	5	6	7	7
Total	90	24	25	39	74	34	64	78	78

Conclusions

- A method for **proving relational properties**
  - translating the property in CHCs
  - transforming the CHCs to better exploit the interaction between predicates
- **Independent of the programming language**
  - The only language specific element is the **interpreter**
  - Can be applied to prove relations between programs that may be written in different programming languages
- **Improves effectiveness** of state-of-the art **CHC solvers**