A general framework for variable aliasing: Towards optimal operators for sharing properties

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Gianluca Amato (Università di Chieti)
Francesca Scozzari (Università di Pisa)

Overview

- We are interested in sharing analysis of logic programs by abstract interpretation
- We look for an optimal operator for computing the mgu in abstract domains which combine sharing and linearity properties.
- We propose a new domain equipped with an optimal operator for unification.
- By abstraction we obtain the optimal operators for King's domain $\mathtt{ShLin}^2$ and $\mathtt{Sharing} \times \mathtt{Lin}$.

The framework

The concrete domain:

$$\texttt{Psub} = \{[\Sigma, U] \mid \Sigma \subseteq \mathit{ISubst}_{\sim_U}, U \in \wp_f(\mathcal{V})\} \cup \{\bot, \top\}$$

A concrete object:

$$\left[\left\{ \left\{ \begin{array}{c} x/f(k) \\ y/g(k) \end{array} \right\}, \left\{ \begin{array}{c} x/f(v) \\ w/y \end{array} \right\} \right\}, \{x, y, w, z\} \right]$$

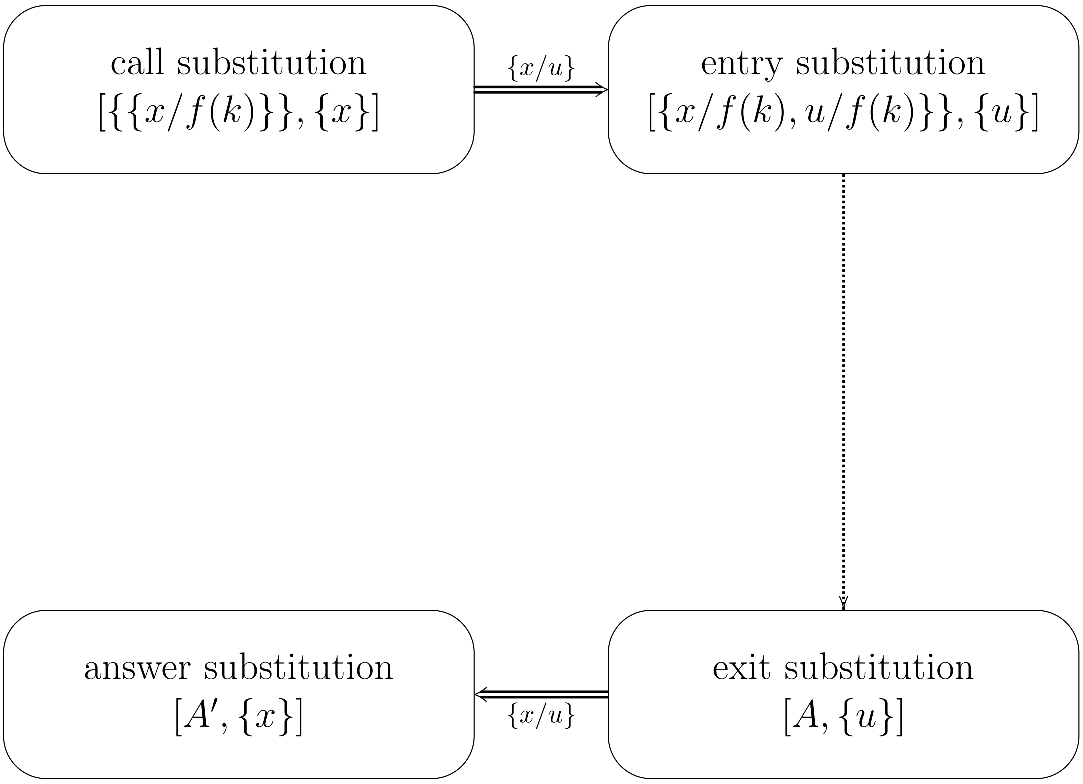
$$\left[\left\{ \left\{ \begin{array}{c} x/f(u) \\ y/g(u) \end{array} \right\}, \left\{ \begin{array}{c} x/f(k) \\ w/y \end{array} \right\} \right\}, \{x, y, w, z\} \right]$$

- The semantics has 3 operators for:
  - (forward and backward) unification
  - projection
  - union
- an operator for (forward) unification unif:

$$\mathsf{unif}_{\mathrm{Ps}}([\Sigma, U], \delta, V) = [\{\mathrm{mgu}(\theta, \delta) \mid \theta \in \Sigma, \}, U \cup V]$$

An example

A goal $p(x)$, a clause $p(u) \leftarrow B$



$$\mathsf{unif}_{\mathrm{Ps}}([\{\{x/f(k)\}\}, \{x\}], \{x/u\}, \{u\})) = [\{x/f(k), u/f(k)\}\}, \{x, u\}]$$

The abstract domain Sharing

$$\texttt{Sharing} = \{[A,U] \mid A \subseteq \wp(U), (A \neq \emptyset \Rightarrow \emptyset \in A), U \in \wp_f(\mathcal{V})\} \cup \{\top, \bot\}$$

$$\left\{ \begin{array}{l} x/f(a,b) \\ y/g(a,b,b,b) \\ z/f(a,a) \end{array} \right\} \Rightarrow^{\alpha} [\{\{x,y,z\},\{x,y\},\emptyset\},\{x,y,z\}]$$

$$[\{\texttt{xyz},\texttt{xy}\},\{x,y,z\}]$$

A more precise domain...

$$\texttt{ShLin}^2 = \Big\{[S,U] \mid S \in \wp_{\downarrow}(Sg^2(U)), U \in \wp_f(\mathcal{V}), S \neq \emptyset \Rightarrow \emptyset \in S\Big\} \cup \{\top, \bot$$

$$\left\{ \begin{array}{l} x/f(a,b) \\ y/g(a,b,b,b) \\ z/f(a,a) \end{array} \right\} \Rightarrow^{\alpha} [\{\texttt{xyz}^{\infty},\texttt{xy}^{\infty}\},\{x,y,z\}]$$

What about unification?
We look for an operator:

$$\mathsf{unif}_X([S,U],\delta,V)$$

which is:

- correct: $\mathsf{unif}_X([S,U],\delta,V)$ approximates $\mathsf{unif}_{\mathrm{Ps}}([\Sigma,U],\delta,V)$
- optimal: $\mathsf{unif}_X([S,U],\delta,V)$ is the best approximation of $\mathsf{unif}_{\mathrm{Ps}}([\Sigma,U],\delta,V))$

A more concrete domain

In order to study the unification operator we move to a more concrete domain: From sets to **multisets**.

$\mathtt{ShLin}^\omega = \{[S, U] \mid U \in \wp_f(\mathcal{V}), S \subseteq \wp_m(U), S \neq \emptyset \Rightarrow \emptyset \in S\} \cup \{\bot_\omega, \top_\omega\}$

$$\left\{ \begin{array}{l} x/f(a,b) \\ y/g(a,b,b,b) \\ z/f(a,a) \end{array} \right\} \Rightarrow [\{\mathtt{xyz}^2, \mathtt{xy}^3\}, \{x, y, z\}]$$

Towards an operator of unification for $\mathtt{ShLin}^\omega$.
multiplicity of an ω-sharing group B in a term t:

$$\chi(B, t) = \sum_{v \in \llbracket B \rrbracket} B(v) \cdot occ(v, t)$$

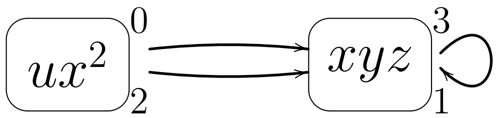
$\chi(x^3yz^4, t(x, y, f(x, y, z))) = 3 \cdot 2 + 1 \cdot 2 + 4 \cdot 1 = 12.$

If B represents the variable v in some substitution θ then $\chi(B, t)$ is the number of occurrences of v in $t\theta$.

The sharing graph

A *sharing graph* $\langle N, l, E \rangle$ is a directed multigraph whose nodes are labeled with sharing groups.

- N is the finite set of nodes
- l is the labeling function from N to sharing groups
- E is the multiset of edges.



A sharing graph is *balanced for the equation* $t_1 = t_2$ if:

1. it is connected;
2. for each node $s \in N$, the out-degree of s is equal to $\chi(l(s), t_1)$ and the in-degree of s is equal to $\chi(l(s), t_2)$.

The above graph is balanced for $x = r(y, y, z)$.
$\chi(ux^2, x) = 2$
$\chi(ux^2, r(y, y, z)) = 0$
$\chi(xyz, x) = 1$
$\chi(xyz, r(y, y, z)) = 3$

The *resultant* ω*-sharing group* of G is the multiset union of the labels.

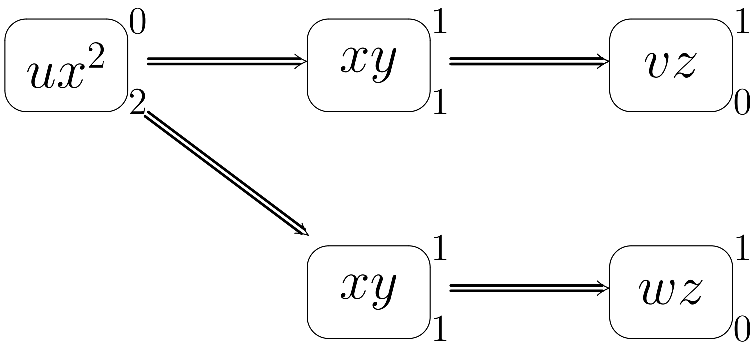
In the above example: $res(G) = ux^3yz$.

From balanced sharing graphs to mgu

A sharing graph represents a possible way to merge together several sharing groups by unifying them with a given binding.

$$\mathrm{mgu}(S, t_1 = t_2) = \{ res(G) \mid G \text{ is a balanced sharing graph for } t_1 = t_2 \text{ with } l(N) \subseteq S \}$$

Let $S = \{ux^2, xy, vz, wz, xyz\}$. The following is a balanced sharing graph for $t(x) = r(y, z)$ and S:



Therefore $uvwx^4y^2z^2 \in \mathrm{mgu}(S, t(x) = r(y, z))$.

We define $\mathrm{mgu}(S, \theta)$ by induction:
$\mathrm{mgu}(S, \epsilon) = \epsilon$
$\mathrm{mgu}(S, \{x/t\} \uplus \theta) = \mathrm{mgu}(\mathrm{mgu}(S, x = t), \theta)$

$$\mathsf{unif}_\omega([S, U_1], \delta, U_2) = [\mathrm{mgu}(S \cup \{\{\!\{v\}\!\} \mid v \in U_2 \setminus U_1\}, \delta), U_1 \cup U_2]$$

unif_ω is optimal (and correct) w.r.t. $\mathsf{unif}_{\mathrm{Ps}}$.

A Characterization for Resultant Sharing Groups

Let S be a set of ω-sharing groups and t_1, t_2 be terms. Then $B \in \mathrm{mgu}(S, t_1 = t_2)$ iff $B = \Uplus_{i \in I} B_i$ where I is a finite set and $\{\!\{B_i\}\!\}_{i \in I} \in \wp_m(S)$ such that:

$$\boxed{\sum_{i \in I} \chi(B_i, t_1) = \sum_{i \in I} \chi(B_i, t_2) \geq |I| - 1}$$

Example.
Consider $S = \{xa, xb, z^2, zc\}$ and the equation $x = z$. For $A = \{\!\{xa, xb, z^2\}\!\}$, we have:

$$\chi(A, x) = 2 = \chi(A, z) \geq |A| - 1$$

Thus $x^2 z^2 ab \in \mathrm{mgu}(S, x = z)$.

For $B = \{\!\{xa, xb, zc, zc\}\!\}$, we have:

$$\chi(B, x) = 2 = \chi(B, z) \not\geq |B| - 1 = 3$$

Actually, $z^2 c^2 x^2 ab \notin \mathrm{mgu}(S, x = z)$.

From $\texttt{ShLin}^\omega$ to $\texttt{ShLin}^2$

The abstraction function from $\texttt{ShLin}^\omega$ to $\texttt{ShLin}^2$ is immediate:

$$\{xy^3, xy^4, xyz, xa, xa^2\} \Rightarrow^\alpha \{xy^\infty, xyz, xa^\infty\}$$

Given $A, B \in \texttt{ShLin}^2$ we define:

$$A \Box B = \lambda v \in \mathcal{V}. A(v) \oplus B(v)$$

where $\oplus$ is defined as

$\oplus$	0	1	∞
0	0	1	∞
1	1	∞	∞
∞	∞	∞	∞

$$\Box\{xy^\infty, xyz, xa^\infty\} = x^\infty y^\infty z a^\infty$$

The multiplicity of a 2-sharing group is the set of multiplicity of its concretization:

$$\chi(A, t) = \{\chi(S, t) \mid \alpha(S) \leq A\}$$

$$\text{mgu}(S, x = t) = \left\{ \Box Y \mid Y \subseteq_m S, \exists n \in \chi(Y, x) \cap \chi(Y, t).\, n \geq |Y| - 1 \right\}$$

A different characterization of mgu

$$
\begin{aligned}
&\mathrm{mgu}(S, x = t) = C^0 \cup \\
&\downarrow\!(\ \{\Box X^2 \mid X \subseteq S_1 \cup S_2, X \cap S_1^{nl} \neq \emptyset, X \cap S_2^{nl} \neq \emptyset\} \cup \\
&\quad \{\Box X^2 \mid X \subseteq S_2^1, X \cap S_1^{nl} \neq \emptyset\} \cup \\
&\quad \{o \Box (\Box X^2) \mid o \in P_1, X \subseteq S_2^1, X \cap S_1^{nl} \neq \emptyset \vee o \in P_1^\infty, X \cap P_2 \neq \emptyset\} \cup \\
&\quad \{\Box X^2 \mid X \subseteq S_1^1, X \cap S_2^{nl} \neq \emptyset\} \cup \\
&\quad \{o \Box (\Box X^2) \mid o \in P_2, X \subseteq S_1^1, X \cap S_2^{nl} \neq \emptyset \vee o \in P_2^\infty, X \cap P_1 \neq \emptyset\} \cup \\
&\quad \{\Box X^2 \mid X \subseteq C^1\} \cup \\
&\quad \{o \Box Y \Box (\Box X^2) \mid o \in P_2, X \subseteq C^1, Y \subseteq_m P_1^1, |Y| = \chi_M(o, t) \in \mathbb{N}^+\})
\end{aligned}
$$

Conclusions

- We propose a new domain $\mathtt{ShLin}^\omega$ as a general framework for investigating sharing and linearity properties.
- We introduce the notion of *(balanced) sharing graph* and provide optimal abstract operators for $\mathtt{ShLin}^\omega$.
- We obtain the optimal operators for forward and backward unification in King's domain $\mathtt{ShLin}^2$ and in $\mathtt{Sharing} \times \mathtt{Lin}$.