Chaotic Behavior of Multidimensional Hamiltonian Systems: Disordered lattices, Granular Chains and DNA models

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Abstract

In this talk I will present results on the chaotic behavior of several disordered, nonlinear Hamiltonian systems, emphasizing the quantification of chaos strength through the computation of the maximum Lyapunov exponent (mLE, see for example [1] and references therein).

Initially, I will discuss the dynamics of the disordered variants of two typical lattice models: the Klein-Gordon oscillator chain and the discrete nonlinear Schrödinger equation. At first, I will explain how one can use symplectic integration schemes for the efficient integration of the equations of motion, as well as the variational equations (needed for the computation of the mLE) of these models [2-7]. Then, I will present results concerning the chaoticity of these systems. In particular, I will focus on the time evolution of the mLE and the distribution of the associated deviation vector. I will emphasize the fact that the observed power law decays of the mLE have exponents different from -1, which is seen in the case of regular motion [8, 9]. This is a clear indication that the dynamics becomes less chaotic, since the constant total energy/norm of the system is shared among more sites, but it does not show any sign of crossing over to regular motion, which could imply a potential halt of spreading. The fact that the same dynamical behaviors are observed for both models signifies the generality of the underlying chaotic mechanisms.

Then, I will present results concerning the energy transport in one-dimensional disordered granular solids. In particular, I will consider the case of a polydisperse granular chain composed of spherical beads of the same material but with radii taken from a random distribution. I will mainly focus on the problem of energy equipartition in these models. I will show that for the highly nonlinear dynamical regime chaos leads to energy equipartition characterized by the same value of the mLE for the same energy, irrespectively of the form of the initial excitation, while in the weakly nonlinear regime it is possible to obtain long lasting energy localization for particular single particle excitations, although the overall dynamics is chaotic [10, 11]. In addition, I will investigate the effect of the discontinuous nonlinearity of the granular chain, which appears whenever neighboring particles are detached, on the chain's chaoticity and energy delocalization [12].

Finally, I will discuss some resent results [13, 14] concerning the effect of heterogeneity, i.e. the percentage and distribution of adenine/thynine (AT) and guanine/cytosine (GC) base pairs, on the chaotic behavior of the Peyrard-Bishop-Dauxois (PBD) DNA model [15]. In particular, I will show the existence of three dynamical behaviors: (i) at low energy densities DNA segments with more AT base pairs are more chaotic, (ii) in a middle energy region the composition of the DNA chain has little impact on its chaoticity, and (iii) at higher energy densities sequences with more GC base pairs appear to be slightly more chaotic, and I will discuss in detail their characteristics.

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