Quantum chaos of generic systems

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Quantum chaos (or wave chaos) is a research field in theoretical and experimental physics dealing with the phenomena in the quantum domain (especially regarding solutions of the Schroedinger equation), or in other wave systems, which correspond to the classical chaos [1,2,8]. These other wave systems are electromagnetic, acoustic, elastic, surface, seismic, gravitational waves, etc. The classical dynamics describes the "rays" of the underlying waves, and the bridge between the classical and quantum mechanics is the semiclassical mechanics, resting upon the short-wavelength approximations. If the classical dynamics is chaotic, we see clear signatures in the quantum (wave) domain, e.g. in statistical properties of discrete energy spectra, in the structure of eigenfunctions, and in the statistical properties of other observables. Quantum chaos occurs in low-dimensional systems, e.g. with just two degrees of freedom (e.g. in 2D billiards), but of course also in multi-dimensional systems. From the above it is obvious that theory and experiment in quantum chaos are of fundamental importance in physics, and, moreover, also in technology.

In generic Hamilton systems we have regions of stable, regular, motion in the classical phase space for certain initial conditions, and chaotic motion for the complementary initial conditions. Accordingly, the corresponding eigenstates are either regular or chaotic, and also the corresponding energy spectra have different statistical properties, namely either Poissonian for the regular eigenstates or the statistics of random matrices in the chaotic case [4,5,6,7,12,13]. In order to decide whether a given eigenstate and the corresponding energy level is regular or chaotic, we must look into the structure of Wigner functions in the "quantum phase space".

Quantum localization of classically chaotic eigenstates is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time t_H of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The

Heisenberg time t_H, as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant $2\pi\hbar$ and the mean spacing between two nearest energy levels ΔE, $t_H = 2\pi\hbar/\Delta E$.

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems, and shall demonstrate the approach and the method in the case of a billiard family in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures [3], using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure A is defined by means of the information entropy, whilst the second one, C, in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measueres are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution $P(S)$ (of nearest neighbours), which at small S is linear $P(S) \propto S$, and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function $P(S) = \exp(-S)$), where there is no level repulsion ($P(0) = 1 \neq 0$). In fully chaotic regime with quantum localization we observe that $P(S)$ at small S is a power law $P(S) \propto S^{\beta}$, with $0 < \beta < 1$. We shall show that there is a functional dependence between the localization measure A and the exponent β, namely that β is a monotonic function of A: in the case of the strong localization are A and β small, while in the case of weak localization (almost extended chaotic states) A and β are close to 1. This presentation includes also our very recent papers on a mixed type billiard system [10, 11, 16] and the stadium billiard [14, 15], as important model systems.

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