COMPLEX DYNAMICS AND STATISTICS OF 1D HAMILTONIAN LATTICES

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In these lectures, I will focus on one class of phenomena, which may be called ``complex'' in the sense that they deviate from what common wisdom expects. Our first discovery is that all types of chaotic behavior *are not qualitatively the same*. Indeed, very close to the boundaries of regular motion, where Lyapunov exponents are small and orbits exhibit ``stickiness'' effects, the statistics of averaged position (or momentum) sums is strongly correlated and probability density functions (pdfs) are not described by pure Gaussians, associated with what we call ``strong chaos'' and Boltzmann Gibbs (BG) statistical mechanics. Instead, the pdfs are well approximated by q (>1) – Gaussians (q=1 being the pure Gaussian), suggesting that their proper description is not through the classical BG entropy S_{BG}, but rather via Tsallis' non-additive (and generally non-extensive) S_q entropy, associated by what one might call ``weak chaos'' [1].

In recent years, generalizations of the so--called FPU -β model were studied, introducing *different ranges of interactions* through a coupling constant that decays as $1/r^{\alpha}$, $0 \leq \alpha < \infty$ ($\alpha \rightarrow \infty$ corresponds to the original nearest neighbor FPU model) [2 - 4]. This led to the remarkable observation that under Long Range Interactions (LRI), $0 \leq \alpha \leq 1$: (i) *complex dynamics* occurs, in the sense that the *maximal Lyapunov exponent* for high specific energies $\varepsilon = E/N$ *decreases* and some type of order is restored, and ii) complex statistics arises, whereby the distribution of time-averaged velocities is well approached by a q(>1)-Gaussian, suggesting that the system is ``weakly chaotic''.

I will also report on more recent findings with J. Macias Diaz and H. Christodoulidi [5,6], which show that LRI influences significantly the important effect of *nonlinear supratransmission* in Hamiltonian 1D lattices. Specifically, we find for the FPU case that *threshold amplitudes increase the longer the interaction ($\alpha \rightarrow 0$)* [5] while for Hamiltonians with KG on – site potentials, there is a sharp decrease of the threshold amplitudes, $0 \leq \alpha < 1.5$ [6], which still remains a mystery!

References

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