A Constructor-Based Reachability Logic for Rewrite Theories

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Outline

Introduction

Origins of Reachability Logic

Reachability Logic (RL) was originally proposed by Rosu et. al to verify programs in $\mathbb{K}$

- based on a rewriting logic (RWL) definition of language $\mathscr{L}$'s semantics
- generalizes both Hoare Logic and Separation Logic
- language-generic: a prover can be generated for each language $\mathscr{L}$ from its rewriting logic semantics $\mathcal{R}_{\mathscr{L}}$

Introduction

From Language-generic to Theory-generic RL

This work addresses the following open problems:

1. Can we develop a reachability logic for general rewrite theories, i.e. $RL(\mathcal{R})$ generalizing $RL(\mathcal{R}_{\mathscr{L}})$?

   ...will allow us to move from verifying *code* to verifying *distributed system designs*

2. How can we *maximize* automation in RL proofs?

3. How can we use RL to prove *invariants* (*invariant paradox*)?

Introduction

From Language-generic to Theory-generic RL

We address questions (1)-(3) via:

1. developing a new RWL-theory based RL *semantics* and very simple *proof system*,
2. utilizing RWL concepts (e.g. *constructors*, *variants*) in our proof system as well as a *equational-theory-generic* SMT solver as a backend based on *variant satisfiability*
3. and applying an appropriate RWL *theory transformation* to prove *invariants*.

The mutual exclusion protocol QLOCK has five rewrite rules:

$$
\begin{array}{lllllllll}
n2w: & < n\ i \mid w & \mid c & \mid q & > & \rightarrow & < n & \mid w\ i \mid c & \mid q\ ;\ i > \\
w2c: & < n & \mid w\ i \mid c & \mid i\ ;\ q > & & \rightarrow & < n & \mid w & \mid c\ i \mid i\ ;\ q > \\
c2n: & < n & \mid w & \mid c\ i \mid i\ ;\ q > & & \rightarrow & < n\ i & \mid w & \mid c \mid q > \\
join: & < n & \mid w & \mid c & \mid q & > & \rightarrow & < n\ i & \mid w \mid c \mid q > \ if\ \phi \\
exit: & < n\ i & \mid w & \mid c & \mid q & > & \rightarrow & < n & \mid w \mid c \mid q >
\end{array}
$$

where $\phi \equiv dup(n\ w\ c\ i) \neq tt$. QLOCK's specification is $\mathcal{R}_{\mathsf{QLOCK}} = (\Sigma, E \cup B, R)$ with R the above rules, B the axioms ACU for _ _ and A for _;_ and, E the equation $dup(s\ s\ s') = tt$.

Outline

Reachability Logic Semantics

Constrained Constructor Patterns

Definition

Let $(\Sigma, B, \vec{E})$ be sufficiently complete w.r.t. constructors Ω.
A *constrained constructor pattern* is a pair: $u \mid \varphi$
such that $u \in T_\Omega(X) \wedge \varphi \in QFForm(\Sigma)$

The set $PatPred(\Omega, \Sigma)$ contains $\bot$ and all constrained constructor patterns, and is closed under $(\vee)$ and $(\wedge)$

The *semantics* of predicate A is $[\![A]\!] \subseteq C_{\Sigma/E,B}$ where:

1. $[\![\bot]\!] = \emptyset$
2. $[\![u \mid \varphi]\!] = \{[(u\rho)!]_{B_\Omega} \in C_{\Sigma/E,B} \mid \rho \in [X \to T_\Omega] \wedge E \cup B \models \varphi\rho\}$.
3. $[\![A \vee B]\!] = [\![A]\!] \cup [\![B]\!]$
4. $[\![A \wedge B]\!] = [\![A]\!] \cap [\![B]\!]$

Definition

Given patterns $u \mid \phi$ and $v_i \mid \psi_i$, a *reachability formula* has form:

$$u \mid \varphi \longrightarrow^{\circledast} \bigvee_i v_i \mid \psi_i$$

Example.

1. $< n \mid w \mid c \mid q > \mid dup(n\ w\ c) \neq tt$
2. $< n \mid w \mid c \mid q > \mid dup(n\ w\ c) \neq tt \longrightarrow^{\circledast}$
   $< n' \mid w' \mid c' \mid q' > \mid dup(n'\ w'\ c') \neq tt$

Reachability Logic Semantics

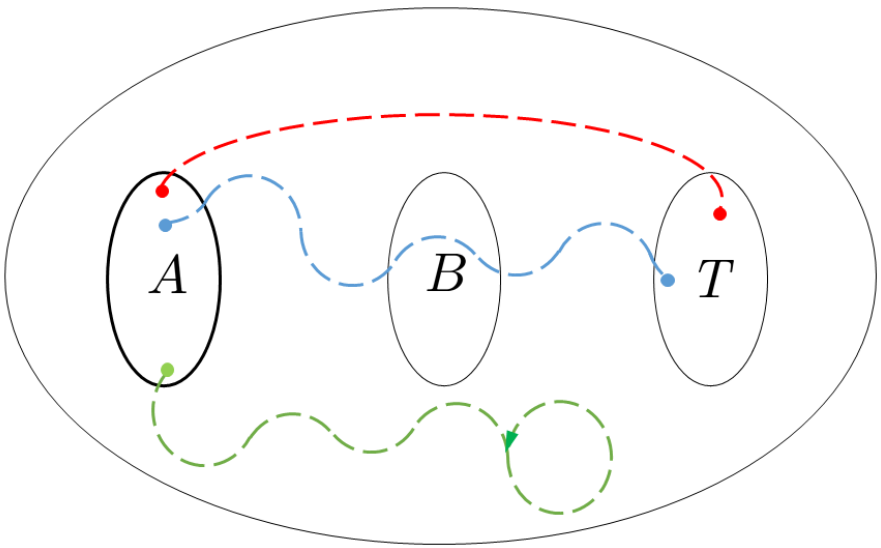
Intuitive Semantics

Q: What does the relation $A \longrightarrow^{\circledast} B$ mean?

A: Suppose we have:

(1) a rewrite theory $\mathcal{R}$
(2) pattern fomulas A, B
(3) and terminating states T

Then $A \longrightarrow^{\circledast} B$ means:
for each state $[t] \in [\![A]\!]$
and rewrite path p from $[t]$,
either: (1) p crosses $[\![B]\!]$ or
(2) p is infinite



- - - indicates counterex.
- - - satisfies $A \rightarrow^{\circledast} B$
- - - *vacuously* satisfies

Reachability Logic Semantics

Formal Semantics

Definition

Let $\mathcal{R} = (\Sigma, E \cup B, R)$ have (a) good *executability conditions*, (b) *constructor* subsignature Ω, (c) and *top sort* $State$ of states. Let $\mathcal{C}_{\mathcal{R}}$ denote the *canonical reachability model*.

$\mathcal{R} \models A \longrightarrow^{\circledast} B$ iff:

For each *concrete state* $[u_0] \in \mathcal{C}_{\mathcal{R},State} \cap [\![A]\!]$ and;
terminating sequence $[u_0] \rightarrow_{\mathcal{R}} [u_1] \cdots [u_{n-1}] \rightarrow_{\mathcal{R}} [u_n]$
There exists $0 \leq j \leq n$ with $[u_j] \in [\![B]\!]$

N.B.: expressible as LTL formula $A \rightarrow (\Box enabled) \vee \Diamond B$

Outline

The Invariant Paradox

Introduction

Recall our example theory QLOCK.

Q: How to express *mutual exclusion* invariant by $A \longrightarrow^{\circledast} B$?

A: Since:

1. $A \longrightarrow^{\circledast} B$ just means $A \rightarrow (\Box enabled) \vee \Diamond B$,
2. and QLOCK is *never terminating*,

then *all* formulas $A \longrightarrow^{\circledast} B$ are satisfied, so we cannot.

(Paradox!!).

The Invariant Paradox

Solving the Invariant Paradox (I)

Recall the structure of the rewrite rules in QLOCK:

$$n2w : < n\ i \mid w \mid c \mid q > \rightarrow < n \mid w\ i \mid c \mid q\ ;\ i >$$

Each rule is topped with a $State$ constructor $< \_ >$. Let's add rule:

$$stop : <\ n \mid w \mid c \mid q\ > \rightarrow [\ n \mid w\ i \mid c \mid q\ ]$$

Note that the $stop$ rule can *terminate* from any state in the combined theory QLOCK-stop.

If $B = \langle \ldots \rangle \mid \varphi$, let $[B]$ denote the predicate $[B] = [\ldots] \mid \varphi$.

Fact. B is an *invariant* from initial states S_0 in QLOCK iff $S_0 \longrightarrow^{\circledast} [B]$ holds in QLOCK-stop.

The Invariant Paradox

Solving the Invariant Paradox (II)

Let $\mathcal{R}$ be a rewrite theory; assume a single $State$ constructor $\langle \_, \ldots, \_ \rangle : w \to State$ and all rules have terms of sort $State$.

Let $\mathcal{R}_{stop}$ extend $\mathcal{R}$ by adding: (i) *fresh* $[\_, \ldots, \_] : w \to State$, and (ii) a *stop rule* $\langle \vec{x} : w \rangle \to [\vec{x} : w]$. Then:

Theorem

B is an invariant for $\mathcal{R}$ from S_0 iff $S_0 \longrightarrow^{\circledast} [B]$ holds in $\mathcal{R}_{stop}$.

Corollary

If $[\![S_0]\!] \subseteq [\![B]\!]$ and $B \longrightarrow^{\circledast} [B]$ holds in $\mathcal{R}_{stop}$, then B is an invariant for $\mathcal{R}$ from initial sates S_0.

Example. Mutual exclusion in QLOCK can be given by $Mutex = \langle n \mid w \mid i \mid i\,;q \rangle \,|\, dup(n\ w\ c\ i) \neq tt \vee \langle n \mid w \mid \emptyset \mid q \rangle \,|\, dup(n\ w\ c) \neq tt$.

Prove: (i) $[\![\langle n \mid \emptyset \mid \emptyset \mid \mathit{nil} \rangle]\!] \subseteq [\![Mutex]\!]$ (ii) $Mutex \longrightarrow^{\circledast} [Mutex]$.

Outline

Inference System

Introduction (I)

Q: Then given RWL theory $\mathcal{R}$, how do we prove $A \longrightarrow^{\circledast} B$?

A: Perhaps surprisingly, two proof rules are enough:

- A rule that traces *rewrite steps* of *symbolic* states in $\mathcal{R}$
- A rule that internalizes *terminating-path-length induction* on $\mathcal{R}$

We call these two rules:

- *Step+Subsumption*
- *Axiom*

Inference System

Introduction (II)

The key ideas are:

1. Proving $A \longrightarrow^{\circledast} B$ may require some *auxiliary lemmas*;
   Let $\mathcal{C}$ denote the formula $A \longrightarrow^{\circledast} B$ plus these lemmas
2. For each formula in $\mathcal{C}$, start with labeled sequents:
   $[\emptyset,\ \mathcal{C}]\ \vdash_T\ u \mid \varphi \longrightarrow^{\circledast} \bigvee_i v_i \mid \psi_i$
3. 1^{st} part ($\emptyset$) is formulas to be assumed as *axioms* (none);
4. 2^{nd} part ($\mathcal{C}$) is formulas to prove that *cannot yet be assumed*
5. the *Step+Subsumption* rule allows us to *inductively assume* $\mathcal{C}$ after a rewrite step with rules $R = \{l_j \to r_j\ if\ \phi_j\}$.

Reachability Logic

Proof Rules (I): Step+Subsumption Rule

$$\frac{\bigwedge_{(j,\alpha)\in \text{UNIFY}(u|\varphi', R)} [\mathcal{A}\cup\mathcal{C},\ \emptyset]\ \vdash_T\ (r_j \mid \varphi' \wedge \phi_j)\alpha \longrightarrow^{\circledast} \bigvee_i (v_i \mid \psi_i)\alpha}{[\mathcal{A},\ \mathcal{C}]\ \vdash_T\ u \mid \varphi \longrightarrow^{\circledast} \bigvee_i v_i \mid \psi_i}$$

with $\varphi' = \varphi \wedge \bigwedge_{(i,\beta)\in \text{MATCH}(u,\{v_i\})} \neg(\psi_i\beta)$ and $R = \{l_j \to r_j \ \mathit{if}\ \phi_j\}$

Note.

- proof rule performs all possible *narrowing* steps with rules R
- goals $u \mid \phi \longrightarrow^{\circledast} B$ with unsatisfiable ϕ are *implicitly removed*

Reachability Logic

Proof Rules (II): The Axiom Rule

$$\frac{\bigwedge_j [\{u' \mid \varphi' \longrightarrow^{\circledast} \bigvee_j v'_j \mid \psi'_j\} \cup \mathcal{A},\ \emptyset] \ \vdash_T \ v'_j\alpha \mid \varphi \wedge \psi'_j\alpha \longrightarrow^{\circledast} \bigvee_i v_i \mid \psi_i}{[\{u' \mid \varphi' \longrightarrow^{\circledast} \bigvee_j v'_j \mid \psi'_j\} \cup \mathcal{A},\ \emptyset] \ \vdash_T \ u \mid \varphi \longrightarrow^{\circledast} \bigvee_i v_i \mid \psi_i}$$

where $\exists \alpha$ with $u =_{E_\Omega \cup B_\Omega} u'\alpha$ and $\mathcal{T}_{\Sigma/E \cup B} \models \varphi \Rightarrow \varphi'\alpha$

Theorem

(Soundness) Let $\mathcal{R}$ be a rewrite theory, and $\mathcal{C}$ a finite set of reachability formulas. If $\mathcal{R}$ proves $[\emptyset,\ \mathcal{C}]\ \vdash_T\ \mathcal{C}$ then $\mathcal{R} \models^{\forall}_T \mathcal{C}$

Outline

Implementation and Case Studies

Reflective Implementation

The proof system has been implemented in *Maude*. Some notes:

1. RWL is *reflective*, Maude's *META-LEVEL* library support was used which supports reasoning over RWL *theories* and *terms*
2. Maude's built-in support for *narrowing* modulo axioms was used to compute successors in the RWL theory $\mathcal{R}$
3. An implementation of a *variant satisfiability*-based, *theory-generic* SMT solver was used to discharge satisfiability and validity proof obligations

Implementation and Case Studies

Case Studies

Example	Description of the System/Property
Choice	Nondeterministically throws away elements from a multiset/eventually only one element left
Comm. Protocol 1	Simple communication protocol/received data is always a prefix of the data to be sent
Comm. Protocol 2	Fault-tolerant communication protocol/all data is eventually received in-order
Dijkstra	Dijkstra's mutual exclusion alg./mutual exclusion
Fixed-Size Token Ring	2-Token ring mutual exclusion alg./mutual exclusion
QLOCK	QLOCK mutual exclusion alg./mutual exclusion
Readers/Writers	Readers-writers mutual exclusion alg./mutual exclusion
Lamport's Bakery	Unbounded Lamport's bakery/mutual exclusion
Thermostat	Open system that dynamically responds to temperature/temperature remains in preset bounds

Implementation and Case Studies

Example Proof Fragment

$$T_1 \equiv \left\{ \frac{}{[\mathcal{C}, \emptyset] \vdash_{[]} [n^3 \mid w^3 \mid \emptyset \mid q^3] \mid dup(n'' \, w' \, p) \neq tt \wedge dup(n^3 \, w^3) \neq tt \rightarrow^{\circledast} [Mutex_1] \vee [Mutex_2]} \; \mathsf{sub}(P_1, \alpha) \right.$$

$$\frac{\dfrac{T_1 \qquad T_2}{\ldots \; [\mathcal{C}, \emptyset] \vdash_{[]} < n'' \mid w' \; p \mid \emptyset \mid q' > \mid dup(n'' \, w' \, p) \neq tt \rightarrow^{\circledast} [Mutex_1] \vee [Mutex_2] \; \ldots} \; \mathsf{axiom}(G_2, \alpha)}{[\emptyset, \mathcal{C}] \vdash_{[]} < n' \mid w' \mid \emptyset \mid q' > \mid dup(n' \; w') \neq tt \rightarrow^{\circledast} [Mutex_1] \vee [Mutex_2]} \; \mathsf{step}(n2w, \theta)$$

where $G_i \equiv Mutex_i \longrightarrow^{\circledast} [Mutex]$, $C \equiv \{G_1, G_2\}$

Outline

Conclusions

We have presented our new theory and implementation of a RL semantics and inference system where:

1. our system is *rewrite-theory-generic*, so it can be applied to analyze distributed system designs
2. our implementation uses a *theory-generic*, *variant satisfiability* SMT solver underneath
3. we applied RWL theory *transformations* in order to specify and verify *invariants*

Future Work

At this point, there are a two clear directions for future work:

1. our *variant satisfiability* implementation currently supports rewrite theories whose equational fragment is decidable—we are developing heuristics for *undecidable* theories
2. we are developing *larger*, *more interesting* case studies to provide further validation for our reachability logic tool

The End

Any Questions?