Scenario Modeling for the Management of International Bond Portfolios*

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Abstract

We address the problem of portfolio management in the international bond markets. Interest rate risk in the local market, exchange rate volatility across markets, and decisions for hedging currency risk are integral parts of this problem. The paper develops a stochastic programming optimization model for integrating these decisions in a common framework. Monte Carlo simulation procedures, calibrated using historical observations of volatility and correlation data, generate jointly scenarios of interest and exchange rates. The decision maker's risk tolerance is incorporated through a utility function, and additional views on market outlook can also be incorporated in the form of user specified scenarios. The model prescribes optimal asset allocation among the different markets and determines bond-picking decisions and appropriate hedging ratios. Therefore several interrelated decisions are cast in a common framework, while in the past these issues were addressed separately. Empirical results illustrate the efficacy of the simulation models in capturing the uncertainties of the Salomon Brothers international bond market index.

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1 Introduction

Does international diversification pay? This question has been answered affirmatively since the 1960's when it was first addressed in an article by Grubel (1968). Several authors have shown that international markets are sufficiently uncorrelated so that portfolio risks can be reduced by a well-diversified international portfolio, even when accounting for the currency exchange risk exposure. This observation is true for both the equities and fixed income markets, and the literature on this topic is extensive, see Michaud et al. (1996), Jorion (1989), and the book by Solnik (1996). Today, however, a simplistic affirmative answer to this question is no more acceptable since many interrelated factors need to be considered. For instance, the volatilities of the fixed income markets—specially bond markets—are typically lower than the volatilities of exchange rates, and hence the currency risk of an international portfolio *might* offset any benefits from diversification, see Kaplanis and Schaefer (1991). Even in the equities markets it has been argued recently that multiple acquisitions in a foreign country will not improve significantly the efficiency of a portfolio when accounting for the added currency risk, see Ziobrowski and Ziobrowski (1995).

The globalization of the financial markets only exacerbates the difficulties facing international bond portfolio managers. On the one hand, reductions in capital controls, technological advances in dissemination of information, and increased computing power for tracking portfolios and forecasting market trends led to an explosive growth of international bond trading over the last decade. On the other hand, the very same developments have led to increased synchronization of market returns. Table 1 summarizes the correlation coefficients between U.S. and foreign bond market (domestic) returns for two different time periods. In all cases, except Canada, the degree of correlation increased with time or remained constant, thus eroding any benefits from international diversification. However, when we account for currency volatilities, and calculate correlation coefficients with market returns converted to USD we observe that the degree of correlation remained constant or even declined. (Data are obtained from Fabozzi (1997, ch. 54).) Hence, benefits can still be reaped from diversification, but those are due primarily to the volatility of exchange rates. Another important question is now raised: Should currency risk be hedged? The answer is intuitively deceptively simple. If currency risk is hedged then international market returns are highly correlated and diversification *may* not pay. If currency risk is not hedged then the lowered correlation of market returns may lead to improved portfolio efficiency. To hedge or not to hedge, becomes now the question, and the answer is "it depends". Kaplanis and Schaefer (1991) show that under some circumstances it is clearly better to be completely hedged, under other circumstances it is clearly better not to hedge, and in yet other cases partial hedging is the optimal strategy. As readers may suspect the most interesting cases, in the sense of being those circumstances that arise in the financial markets, are those that require partial hedging. The benefits of optimal hedg-

Market	**Domestic returns**		**USD returns**
	1978–1995	1990–1995	1990–1995
Australia	.32	.68	.42
Canada	.71	.51	.49
France	.34	.49	.38
Germany	.50	.51	.33
Japan	.38	.43	.26
Netherlands	.52	.53	.35
Switzerland	.36	.46	.26
U.K.	.39	.49	.39
Intnl. index	.59	.64	.44

Table 1: Correlation coefficients between US and other international bond markets computed using both domestic returns and returns converted to US dollars.

ing ratios however are "far from clear" (Kaplanis and Schaefer). Indeed both practitioners (Fabozzi, 1997, chs. 54–55) and academics (Jorion, 1989) agree that hedging should be done on a selective basis when conditions warrant it and the hedging policy may be dependent on the asset mix.

In the setting of the complex interactions between market returns and exchange rate volatilities it becomes essential for a portfolio manager to have at his disposal a systematic methodology for analyzing the interrelated decisions of (*i*) asset allocation among the international markets, (*ii*) bond-picking in each market, and (*iii*) optimal level of hedging. Recently, Mulvey and Zenios (1994) argued that portfolios of bonds should be managed based on the ideas of diversification, looking at the co-movements of the securities under various plausible economic scenarios, instead of using portfolio immunization with simple risk measures such as duration or convexity matching. The ideas of diversification were applied successfully to the management of mortgage-securities portfolios (Golub et al., 1995), callable bond portfolios (Vassiadou-Zeniou and Zenios, 1996) and high-yield bonds (Mulvey and Zenios, 1994). This framework is particularly suitable for the management of international bond portfolios when the three interrelated decisions outlined above depend on scenarios of local market returns and exchange rates.

Stochastic programming models provide the appropriate mathematical framework for optimizing the portfolio decisions under the generated scenarios. These models are multiperiod optimization models for planning under uncertainty based on plausible scenarios for the realization of the uncertain data. Their origins go back to the 1950's with the work of Dantzig (1955) and Wets (1966) and have recently been gaining widespread acceptance for modeling financial decision making problems. See, e.g., Mulvey and Vladimirou (1992), Dembo (1993), Carino et al. (1994), Carino, Myers and Ziemba (1998), Carino and

Ziemba (1998), Golub et al. (1995), Koskosides and Duarte (1997), the recent textbooks by Kall and Wallace (1994) and Censor and Zenios (1997), or the volume by Ziemba and Mulvey (1998).

The adoption of scenario based methodology is particularly suitable in view of anticipated changes in the global financial markets. For instance, the introduction of the EURO in a unified European financial market raises the issue of credit risk for the member countries. As currently no market data are available to calibrate credit risk volatilities the use of scenarios provides the only viable approach for hedging this type of risk based on plausible scenarios of credit risk premia that can be inferred from economic theory (e.g., based on interest rate differentials).

In this paper we develop integrated simulation and optimization models for the management of international bond portfolios. The portfolio management philosophy we adopt is that of *indexation*, i.e., developing a basket of assets that tracks the returns of a broadly defined market index; the Salomon Brothers international index is our target. The problem setting and the general modeling approach we take are described in section 2. The stochastic programming model that jointly determines asset allocation, bond-picking, and hedging decisions is developed in section 2.1. Scenario generation models (section 3) generate joint scenarios of market returns and exchange rates. Some preliminary results (section 4) with the backtesting of the scenario generation models illustrate their efficacy in generating scenarios that encompass the true market changes. The performance of the portfolio optimization models on these scenarios is the topic of ongoing investigations jointly with a Swiss bank.

2 Integrated simulation and optimization models for tracking an international bond index

We consider the problem of a portfolio manager whose mandate is to manage a bond portfolio in a way that it tracks a broadly defined international market index. Indexed fixed-income funds have gained in popularity over the last decade for reasons that are well documented in the literature, see, e.g., Fabozzi (1997, ch. 47), and we adopt this particular management philosophy for developing the models.

A bond index in each market $i = 1, 2, \ldots, m$, is constructed by creating a representative sample Φ_i of size N_i from the universe of eligible bonds Ω_i. For each security $j = 1, 2, \ldots, N_i$, in the representative set, the index specifies its relative weight ϕ^i_j which reflects the capitalization structure of the universe set Ω_i with bonds that have characteristics identical or similar to the jth bond. The global bond index is represented by a set Γ of country indices and the relative weights γ_i assigned to the bond index of each country based on the market value of the different indices. These weights are a measure of the share of the bond market of the ith country in the world bond market.

In practice the sets Ω_i may consist of thousands or even hundreds of thousands of bonds (differing by issuer, issue data, maturity date, coupon payments etc), while the representative sets Φ_i consist of hundred or so representative securities. Global bond indices, such as the Salomon Brothers Global index, consist of holdings in the bond markets of the major industrialized nations (i.e., $m = 7$ for the G7 index).

The manager of an international bond portfolio must determine the fraction of the portfolio value invested in each of the m markets, and to pick specific bonds from each market Ω_i to add to the portfolio. These decisions are usually made in two steps. An asset allocation committee determines first the exposure of the portfolio to each market, and then traders identify mispriced bonds in each market and construct the country-specific portfolio. This portfolio has to track the country specific index as well, so constraints on duration, maturity etc may be imposed on the trader. Finally, once the country-specific funds are constructed the currency exposure may be hedged.

We denote the weights allocated to the index of each country $i = 1, 2, \ldots, m$, by Z_{A_i}, and the vector of weights of the bonds in each country by $Z_D^i = (Z_{D_j}^i)_{j=1}^{N_i}$; the subscripts A and D denote that decisions are aggregated and disaggregated by market, respectively. The model we develop in the next section is a multiperiod model, and therefore the variables are time-dependent. Furthermore, decisions made after the first time period are conditioned on the realized returns of the market indices and exchange rates, hence the variables are also scenario dependent. The dependencies of the variables on time and scenarios are made precise later. For now we use the simplified notation to introduce the problem and illustrate the modeling process in Figure 1.

One of the key contributions of this paper is in developing models for integrating the three financial decisions—asset allocation, bond-picking, and currency hedging—in a common framework. To what extend this integration pays is the topic of our empirical investigations, that have not yet been completed. However, the significance of integrating several interrelated financial decisions in a common framework finds widespread acceptance in the finance literature, see, e.g., Merton (1990), and Holmer and Zenios (1995).

Having set the stage of the portfolio manager's problem in the previous section we now define the stochastic programming models. First, we give a generic description of a two-stage stochastic programming model for constructing a portfolio of m assets to track an index. We then specialize this model to solve independently the asset allocation and the bond-picking problems for international bond portfolio management. The third model integrates the asset allocation and bond-picking decisions. Finally, we incorporate hedging decisions in the models.

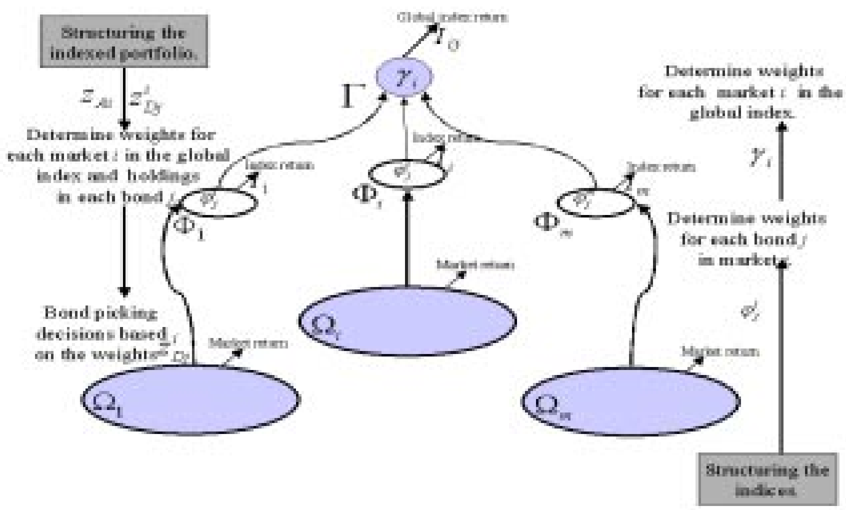


Figure 1: The process for structuring the market indices and the indexed portfolio.

2.1 Canonical stochastic programming model for tracking an index

2.1.1 Notation

The model is developed using cashflow accounting equations and inventory balance equations for each asset category. It is customary in asset allocation models to make decisions in terms of percentage of wealth invested in each asset class. However, since our models deal with fixed-income securities and allow for portfolio rebalancing, we measure investment decisions in dollars of face value so that we can account for price changes, transaction costs and exchange rate fluctuations. In the canonical model below there is only a single currency, but in the integrative model we need to make the distinction between the *base* currency and the *domestic* currency; the former is the currency in which our investor measures her return, and the latter is the currency of the country in which some investments are made. Conversion between domestic and base currency are made using the appropriate exchange rate.

Define first parameters of the model at periods $t = 1, 2, \ldots, T$:

S_t: sets of scenarios anticipated at time t. We use s_t to index scenarios from the set S_t. Let l_t denote paths of scenarios that are resolved (i.e., all information becomes known) until period $t = 1, 2, \ldots, T$. Paths are denoted by $l_t = (s_0, \ldots, s_{t-1})$, and with each path we associate a probability π_{l_t}. We also let P_t denote the set of all paths that can be constructed by combining scenarios from the scenario sets $S_0, S_1, \ldots, S_{t-1}$. Note that l_t denotes all information that becomes known by instance t, while s_t denotes scenarios anticipated at t. Paths are not defined for $t = 0$ since all information is assumed known at this time instance.

M: set of available asset classes, with cardinality m.

c_0: initial endowment in the riskless asset.

b_0: vector denoting the composition of the initial portfolio.

ζ_0: vector of bid prices at $t = 0$. These prices are known with certainty.

$\zeta_t(l_t)$: vector of bid prices realized at t. These prices depend on the path of scenarios followed from 0 to t.

$k_t(l_t, s_t)$: vector of *cash accrual* factors during the interval $[t, t+1]$. These factors indicate cash generated per unit holdings in the asset class due to coupon payments. We define $k_t(l_t, s_t) \doteq k_0(s_0)$ for $t = 0$, where $k_0(s_0)$ is given conditioned on the scenarios anticipated at $t = 0$, but not on any path since all information prior to $t = 0$ is assumed known.

$\rho_t(l_t)$**:** short term riskless reinvestment rate at period t. These rates depend on the path followed up to t (typically interest rates depend only on s_{t-1} only and not on the whole path l_t).

$e_t(l_t)$**:** vector of exchange rates between the domestic and the base currency. Exchange rates are scenario dependent.

$I_T(l_T)$**:** return of the index to be tracked; this value depends on the path and is computed based on price appreciation (or depreciation) of the securities in the index plus any accrued cashflow from reinvested coupon payments. The return of the index of the ith market, in the domestic currency, is given by

$$I_T(l_T) \doteq I_{T_i}(l_T) = \frac{\sum_{j=1}^{N_i} \left(\phi_j^i \zeta_{Tj} + \rho_{T-1} \phi_j^i k_{T-1_j} \right) - V_{T-1_i}}{V_{T-1_i}}, \qquad (1)$$

where V_{T-1_i} is the value of the ith country index at the previous time period and is given recursively by $V_{t_i} = V_{t-1_i} I_t(l_t)$ where V_{0_i}, the initial value of the index, is assumed given.

δ**:** transaction costs. We assume, for simplicity and without loss of generality, that transactions costs are incurred only when buying assets and that they are identical for all types of securities.

Now define decision variables. We have four distinct decisions at each point in time: how much of each asset class to buy, sell, or hold in the portfolio, and how much to invest in the riskless asset. All variables are constrained to be nonnegative, therefore no short sales or borrowing are allowed.

First stage variables at $t = 0$. These are decisions made at the beginning of the planning horizon, when market information is completely specified

X_0**:** vector denoting the face value bought of each asset class.

Y_0**:** vector denoting the face value sold of each asset class.

Z_0**:** vector denoting the holdings in the portfolio.

v_0**:** amount invested in the riskless asset.

Time-staged variables at some future point in time $t = 1, 2, \ldots, T$, conditioned on the path followed up to that time instance.

$X_t(l_t)$**:** vector denoting the face values bought of each asset.

$Y_t(l_t)$**:** vector denoting the face values sold of each asset.

$Z_t(l_t)$**:** vector denoting the holdings in the portfolio.

$v_t(l_t)$**:** amount invested in the riskless asset.

A word on our use of subscripts and superscripts. Superscripts are used to denote vectors, and subscripts denote elements of a vector. For instance, ζ^i denotes a vector of prices for instruments in the ith asset class, ζ_i denotes the price of the ith asset, and ζ^i_j denotes the price of the jth instrument from the ith asset category.

2.1.2 Model formulation: First-stage constraints

At the first stage (i.e., at time $t = 0$) all prices are known with certainty. The *cashflow accounting* equation specifies that the original endowment in the riskless asset, plus any proceeds from liquidating part of the existing portfolio, equals the amount invested in increasing the holdings in some of the asset categories plus the amount invested in the riskless asset:

$$c_0 + \sum_{i=1}^{m} \zeta_{0_i} Y_{0_i} = \sum_{i=1}^{m} (\zeta_{0_i} + \delta) X_{0_i} + v_0. \tag{2}$$

For each asset in the portfolio we have an *inventory balance* constraint:

$$b_{0_i} + X_{0_i} = Y_{0_i} + Z_{0_i} \text{ for all } i \in M. \tag{3}$$

2.1.3 Model formulation: Time-staged constraints

Decisions made at any time period $t = 1, 2, \ldots, T$, after $t = 0$ depend on the path l_t. Hence, we have one constraint for each path. These decisions also depend on the investment decisions made at the previous time period. Note that for $t = 0$ these variables are independent of any path, and the argument l_t is superfluous. To simplify the notation we drop the arguments l_t and s_t from all variables and parameters below. It is understood that each time-indexed variable or parameter depends on these arguments as specified in section 2.1.1.

Cashflow accounting limits the increase in holdings of the asset classes and the riskless asset to be equal to the income generated from the existing portfolio during the holding period, plus any cash generated from sales. There is one constraint for each path $l_t \in P_t$:

$$\begin{aligned} \rho_{t-1} v_{t-1} \quad &+ \quad \sum_{i=1}^{m} k_{t-1_i} Z_{t-1_i} + \sum_{i=1}^{m} \zeta_{t_i} Y_{t_i} \\ &= \quad \sum_{i=1}^{m} (\zeta_{t_i} + \delta) X_{t_i} + v_t. \end{aligned} \tag{4}$$

Inventory balance equations constrain the amount of each asset sold or remaining in the portfolio to be equal to the outstanding amount at the end of the holding period, plus any additional amount purchased. There is one constraint for each asset $i \in M$ and for each path $l_t \in P_t$:

$$Z_{t-1_i} + X_{t_i} = Y_{t_i} + Z_{t_i}. \tag{5}$$

2.1.4 Calculation of return of the portfolio

At the end of the planning horizon T and for each realized path l_T we calculate the return of the portfolio. This value depends on the composition of the portfolio and the value of the assets at T, and on any cash carried over from previous periods. (The value of an asset category is the return of the asset, i.e., the market value per unit asset.) The return of the portfolio is given by:

$$R_p(l_T) \doteq R_p(Z_T(l_T)) = \frac{v_T + \sum_{i=1}^m \zeta_{T_i} Z_{T_i} - V_{p0}}{V_{p0}}, \tag{6}$$

where V_{p0} is the initial value of the portfolio.

2.1.5 Objective function

The objective function maximizes the expected utility of excess return of the portfolio over the index,

$$\text{Maximize} \sum_{l_T \in P_T} \pi_{l_T} \mathcal{U}\left(\frac{R_p(l_T)}{I_T(l_T)}\right), \tag{7}$$

where $\mathcal{U}(\cdot)$ denotes the utility function. Note that one could adopt a function maximizing the expected utility of total return, thus solving a global asset allocation model.

We choose to maximize a utility function of excess return to allow for tradeoffs of growth versus security in the context of the dynamic financial decision-making problem faced by our investors, see, e.g., MacLean, Ziemba and Blazenko (1992). Choosing for instance a logarithmic utility function we can implement in our model the investors' wish to follow a growth optimal strategy over the long run. For now we do not specify any particular form for the utility function; see Ingersoll (1987) for a discussion of utility optimization models and Hakansson and Ziemba (1995) for capital growth theory.

2.2 Aggregated models for tracking an international fixed income index

We apply now the canonical model developed above to the problem of tracking an international fixed income index. We first develop a model for determining the allocation of the investor assets among the m market indices (global asset allocation model). Then we formulate the model for determining the bond-picking decisions in each market (bond-picking model).

2.2.1 Global asset allocation model

The decision variables of the canonical model X, Y and Z are now aggregate variables, that is, $X \doteq X_A = (X_{A_i})_{i=1}^m$, $Y \doteq Y_A = (Y_{A_i})_{i=1}^m$ and $Z \doteq Z_A = (Z_{A_i})_{i=1}^m$. Time-staged variables also carry the appropriate time and scenario arguments as defined in section 2.1.1. The set of assets M is the set of country indices Γ, and the index to be tracked is the global market index. The return of the global index is given by

$$I_T(l_T) \doteq I_G = \sum_{i=1}^{m} \gamma_i e_{T_i} I_{T_i}(l_T), \tag{8}$$

where $I_{T_i}(l_T)$ is the return of the ith country index expressed in the domestic currency (cf. eqn. (1).

The stochastic programming model for the global asset allocation problem is simply a re-statement of the equations in section 2.1 using the aggregated variables and tracking the global index given by equation (8). The optimal value $Z^*_{A0_i}$ at the first-stage, for $i = 1, 2, \dots, m$, denotes the allocation of assets to the ith market.

2.2.2 Bond-picking model

Once the asset allocations have been determined for each market we can solve the bond-picking model. The model can be written by modifying the model in section 2.1 as follows: The decision variables, for each market $i = 1, 2, \dots, m$, are the disaggregated variables as defined in section 2. That is, $X \doteq X_D^i = (X^i_{D_j})_{j=1}^{N_i}$, $Y \doteq Y_D^i = (Y^i_{D_j})_{j=1}^{N_i}$, and $Z \doteq Z_D^i = (Z^i_{D_j})_{j=1}^{N_i}$. Time-staged variables also carry the appropriate time and scenario indices. The initial endowment (cash c_0) in the ith market is $Z^*_{A0_i}$, i.e., the optimal solution of the global asset allocation model of section 2.2.1. The index to be tracked is the index of the ith market with return given in the domestic currency by (1).

The stochastic programming program for the bond-picking problem is simply a re-statement of the equations in section 2.1 using the disaggregated variables, and tracking the country index as given by equation (1). The optimal solution $(z^{i*}_{D0_j})_{j=1}^{N_i}$, at the first-stage denotes the holdings in each of the j bonds of the ith market.

2.3 Integrative model for tracking an international fixed income index

The problems described by the asset allocation and bond-picking models (sections 2.2.1 and 2.2.2) novel can be solved with methods which differ from those we have described. The interesting contribution of our modeling approach is that these two models can be combined to solve the asset allocation and bond-picking problems in an integrated fashion.

We formulate the model in detail next, but first we give a brief sketch. The model uses the disaggregated variables adopted in the bond-picking model (section 2.2.2), but it tracks the global index I_G which is an aggregate of the m market indices as given in equation (8). The model specifies optimal bond-picking decisions in each one of the m markets to track the global index. The asset allocation decisions are then computed as the aggregate of the total exposure to each market (i.e., $Z^*_{A_i} = \sum_{j=1}^{N_i} Z^{i*}_{D0_j}$, where $Z^{i*}_{D0_j}$ is the optimal holding in the jth bond of the ith market at the first time period $t = 0$). In this model we incorporate exchange rates since assets may be invested in different currencies.

We can now write down the multi-stage stochastic programming model with model parameters as defined in section 2.1.1 and using the disaggregated variables.

2.3.1 Integrative model: First-stage constraints

The first stage (i.e., at time t_0) cashflow accounting equation of the disaggregated model is:

$$c_0 + \sum_{i=1}^{m} e_{0_i} \sum_{j=1}^{N_i} \zeta^i_{0_j} Y^i_{D0_j} = \sum_{i=1}^{m} e_{0_i} \sum_{j=1}^{N_i} (\zeta^i_{0_j} + \delta) X^i_{D0_j} + v_0. \qquad (9)$$

The inventory balance constraint is:

$$b^i_{0_j} + X^i_{D0_j} = Y^i_{D0_j} + Z^i_{D0_j} \text{ for all } i \in \Gamma \text{ and } j \in \Phi_i. \qquad (10)$$

Note that there is a single cashflow accounting equation for the base currency. The model could be easily extended to incorporate cashflow accounting equations for each domestic currency, and model exchange rates in transferring funds from one currency to another. For the sake of simplicity we assume here that all sales and purchases are made into and from the base currency, thus avoiding the need to keep separate cashflow variables.

2.3.2 Integrative model: Time-staged constraints

Cashflow accounting constraints of the disaggregated model at any time period t after $t = 0$ depend on the path l_t. These constraints limit the increase in holdings for each bond in each market and in the riskless asset to be equal to the income generated from the existing portfolio during the holding period, plus any cash generated from sales. Note that for $t = 0$ the decision variables are independent of any path, and the argument l_t is superfluous.

There is one constraint for each path $l_t \in P_t$ (the arguments l_t are dropped from all variables and parameters below for simplicity of notation):

$$\rho_{t-1} v_{t-1} \quad + \quad \sum_{i=1}^{m} e_{t_i} \sum_{j=1}^{N_i} k^i_{t-1_j} Z^i_{Dt-1_j} + \sum_{i=1}^{m} e_{t_i} \sum_{j=1}^{N_i} \zeta^i_{t_j} Y^i_{Dt_j}$$

$$= \sum_{i=1}^{m} e_{t_i} \sum_{j=1}^{N_i} (\zeta^i_{t_j} + \delta) X^i_{Dt_j} + v_t. \qquad (11)$$

Inventory balance equations constrain the amount of each bond sold or remaining in the portfolio to be equal to the outstanding amount at the end of the holding period, plus any additional amount purchased. There is one constraint for each bond and for each path $l_t \in P_t$:

$$Z^i_{Dt-1_j} + X^i_{Dt_j} = Y^i_{Dt_j} + Z^i_{Dt_j} \text{ for all } i \in \Gamma,\ j \in \Phi_i. \qquad (12)$$

2.3.3 Integrative model: objective function

At the end of the planning horizon T and for each path $l_T \in P_T$ we calculate the return of the portfolio. This value depends on the composition of the portfolio and the value of the bonds at T and on any accrued cashflow from previous periods. The return of the portfolio is given by

$$R_p(l_T) \doteq R_p(Z_{DT}(l_T)) = \frac{v_T + \sum_{i=1}^{m} e_{T_i} \sum_{j=1}^{N_i} \zeta^i_{T_j} Z^i_{DT_j} - V_{p0}}{V_{p0}}, \qquad (13)$$

where V_{p0} is the initial value of the portfolio. The objective function maximizes the expected utility (cf. eqn. (7)) of excess return of the portfolio (cf. eqn. (13)) over the global index (cf. eqn. (8)).

2.3.4 Optimal currency hedging ratios

We incorporate now hedging decisions in the optimization model. We define a new variable $(H_{0_i})_{i=1}^m$ to denote the amount of each currency hedged at agreed upon 1-period forward rates $(f_{0_i})_{i=1}^m$ at period $t = 0$. Let also $(H_{t_i}(l_t))_{i=1}^m$ denote the amount hedged at forward rates $(f_{t_i}(l_t))_{i=1}^m$ at period t under path l_t.

The cashflow accounting constraints must be modified to account for the fact that at each period t an amount $H_{t_i}(l_t)$ of the i currency will be exchanged at rate $f_{t_i}(l_t)$ and any remaining amount will be exchanged at the current exchange rate $e_{t_i}(l_t)$. Recall that there is one constraint for each path $l_t \in P_t$, and that the arguments l_t are dropped from all variables and parameters below for simplicity of notation.

The total cashflow in the ith currency—inflows from coupon payments and security sales and outflows from security purchases—at period t under scenario l_t is given by

$$w_{t_i} = \sum_{j=1}^{N_i} k^i_{t-1_j} Z^i_{Dt-1_j} + \sum_{j=1}^{N_i} \zeta^i_{t_j} Y^i_{Dt_j} - \sum_{j=1}^{N_i} (\zeta^i_{t_j} + \delta) X^i_{Dt_j} \qquad (14)$$

The cashflow accounting quation (11) is rewritten to incorporate hedging as:

$$\rho_{t-1} v_{t-1} + \sum_{i=1}^{m} \left(f_{t-1_i} H_{t-1_i} + e_{t_i} (w_{t_i} - H_{t-1_i}) \right) = v_t. \qquad (15)$$

At the end of the planing horizon the return of the portfolio—in the base currency—will be a function of the amount hedged and the forward and current exchange rates. The return calculation takes the form:

$$R_p(l_T) \doteq R_p(Z_{DT}(l_T)) = \qquad (16)$$
$$\frac{v_T + \sum_{i=1}^{m} \left(f_{T-1_i} H_{T-1_i} + e_{T_i} \left(\sum_{j=1}^{N_i} \zeta_j^i Z_{DT_j}^i - H_{T-1_i} \right) \right) - V_{p_0}}{V_{p_0}}.$$

The integrative model that jointly determines asset allocation, bond-picking, and hedging decisions is the model of section 2.3 with the modified return calculation and cashflow equations given in this section.

3 Monte Carlo scenario generation

In order to implement the portfolio optimization programs we need a simulation procedure to generate interest rate and exchange rate scenarios. Once such scenarios are generated the calculation of bond prices—in both domestic and base currency—conditioned on the observed interest rates is straightforward; see, e.g., Mulvey and Zenios (1994). The model provides a structural interpretation of bond returns: returns are not simply forecasted on the basis of information variables; instead they are deduced from the scenarios of interest rates.

Models for jointly estimating interest and exchange rates are prevalent in the finance literature. However, the problem of jointly modeling these processes in a way that is consistent with market observations is a difficult one and the success of various models in accurately predicting interest and exchange rate changes has been limited. Meese and Rogoff (1983) compared the predictive abilities of a variety of exchange rate models. Their key result was that no existing structural exchange rate model could reliably out-predict the naive alternative of a random walk at short- and medium-run horizons, even when aided by actual future values of the regressors. This extremely negative finding has never been entirely convincingly over-turned despite many attempts, see Frankel and Rose (1995). The simple random walk model of the exchange rate has become the standard benchmark for empirical exchange rate performance, no matter how uninteresting it is per se.

We adopt in this paper a Monte Carlo simulation procedure that encompasses two methodologies based on recent finance literature that are widely accepted by practitioners. Interest rates are generated using binomial lattice models of the term structure, see, e.g., Black, Derman and Toy (1990). Joint scenarios of interest and exchange rates are generated using the Value-at-Risk methodology, see RiskMetrics (1996, ch. 7). The scenario generation procedure uses real probabilities based on RiskMetrics data to generate the scenarios that are input to the optimization model and risk-neutral probabilities from the binomial lattice to generate further scenarios that are used for bond pricing.

3.1 Interest rate scenario generation

Interest rates are generated from a binomial lattice. The short-term rates for time period t are assumed to be lognormally distributed, and the t-period spot rate implied from the short-term rates is assumed to have a mean value equal to the t-period spot rate given by the current term structure of interest rates, and volatility equal to the t-period volatility of interest rates. Assuming further a discretized binomial approximation one obtains a model such as the one given by Black, Derman and Toy (1990) whereby the short-term rate at period t and at the sth state of the binomial lattice is given by:

$$r_t^s = r_t^0 (k_t)^s, \qquad (17)$$

where $\{r_t^0\}_{t=0}^T$ and $\{k_t\}_{t=0}^T$ are parameters of the binomial lattice estimated using the procedures described by Black, Derman and Toy (1990), and all states of the lattice are equiprobable; the lattice describes a risk-neutral world. A Black-Derman-Toy lattice is calibrated for each currency separately using the term structure of interest rates and the term structure of volatility for each country.

3.2 Exchange rate scenario generation

Interest rates in the two countries are key determinants of the exchange rate between the currencies. Hence, we adopt an approach whereby exchange rate scenarios are conditional on the interest rates of the two currencies, the base currency and the foreign currency. We assume that the logarithms of the ratios of exchange rates at period t to period $t-1$, and the logarithms of the ratios of spot interest rates at period t to period $t-1$ follow a multivariate normal distribution. This is a standing assumption in RiskMetrics and the Value-at-Risk methodology, and it is also well justified in the context of our models[1].

We denote the logarithms of the ratios of all random variables by the p-dimensional random vector ω, where $p = mT + (m-1)$. The dimension of ω is equal to the number of currencies times the number of time periods—for the spot rate random variables—plus number of currencies minus 1—for the

[1]Daily and weekly rates do not follow normal distributions; however, there is lack of empirical evidence against normality for monthly data such as those used in our model.

exchange rates of each currency against the base currency. The real probability density function of ω is given by

$$f(\omega) = (2\pi)^{-p/2} \mid \Sigma \mid^{-1/2} \exp\left[-\frac{1}{2}(\omega-\mu)'\Sigma^{-1}(\omega-\mu)\right], \qquad (18)$$

where μ is the expected value of ω and Σ is the covariance matrix.

The covariance matrix Σ and the expected values μ of all random variables involved in the model are available on a daily basis by RiskMetrics and can be used to build the multinormal distribution. Once the multivariate normal distribution is built we can use it in Monte Carlo simulations, using either the standard Cholesky factorization approach (see, e.g., RiskMetrics, 1996, ch. 7) or the scenario generation procedures based on principal component analysis discussed in Jamshidian and Zhu (1997). We do not use any one of these approaches in our simulations, since we have additional information on the interest rate random variables from the binomial lattice which we would like to incorporated in our sampling of the multinormal distribution.

We partition the multivariate normal variable ω into two subvectors ω_1 and ω_2, where ω_1 is the vector of dimension $p_1 = mT$ of random variables corresponding to interest rates, and ω_2 is the vector of dimension $p_2 = m-1$ of exchange rates. The expected value vector and covariance matrix are partitioned similarly as

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}. \qquad (19)$$

It is known (see, e.g., Jobson, 1992, ch. 7) that the marginal probability density function of ω_2 given $\omega_1 = \omega_1^*$ is

$$f(\omega_2 \mid \omega_1 = \omega_1^*) = (2\pi)^{-p_2/2} \mid \Sigma_{22.1} \mid^{-1/2} \exp\left[-\frac{1}{2}(\omega_2-\mu_{2.1})'\Sigma_{22.1}^{-1}(\omega_2-\mu_{2.1})\right], \qquad (20)$$

where the conditional expected value and covariance matrix are given by

$$\mu_{2.1}(\omega_1^*) = (\mu_2 - \Sigma_{21}\Sigma_{11}^{-1}\mu_1) + \Sigma_{21}\Sigma_{11}^{-1}\omega_1^*, \qquad (21)$$

and

$$\Sigma_{22.1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}, \qquad (22)$$

respectively. Exchange rate scenarios for period t conditioned on value of the interest rates vector given by ω_1^* can be generated from today's exchange rates and the multivariate normal variables from (20) through the expression

$$e_{t_i} = e_{0_i} e^{\sigma_i \sqrt{t}\omega_{2_i}},$$

where e_{0_i} is today's exchange rate for the i currency, σ_i is the 1-period volatility of the ith exchange rate, and ω_{2_i} is the conditional multivariate random variable corresponding to the ith exchange rate.

3.3 Sampling scenarios

The procedures described above allow us to generate a large number of scenarios of both interest rates and exchange rates. We describe here our scenario sampling procedure. First, we note that the time horizon for the indexation model is of the order of a few months. During this time interval we generate exhaustively all interest rate scenarios from the binomial lattice. For instance, for a 3-month horizon we have 8 interest rate paths for each currency. The probability mass functions for all possible combinations of these paths (e.g., $8 \times 8 \times 8$ for a three currency model) are calculated by integrating the multivariate normal probability density function (18) using the covariance matrix of interest rates as obtained, for example, by RiskMetrics. Which scenarios from the large number generated ($8 \times 8 \times 8$ in this example) do we include in the scenario optimization model? We select a small fraction (around 1/10th of the total number of scenarios) of the extreme scenarios, that is scenarios whereby the interest rates in one or more currency take an extreme path of the binomial lattice such as "all-up" or "all-down". We also select an additional 1/2 of the total number of scenarios from those scenarios that are the most likely.

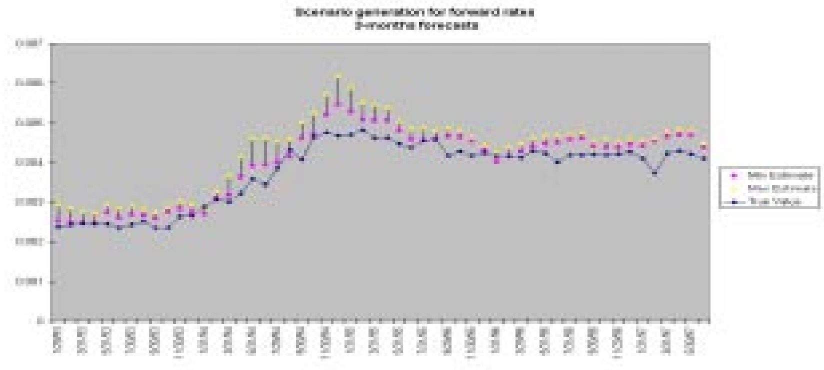
Having thus generated interest rate scenarios we can condition on these values, and sample equation (20) to obtain exchange rate scenarios. We sample ten exchange rate scenarios for each tuple of interest rate scenarios.

4 Some empirical results

The simulation models described in the previous section were tested on a set of data obtained from the Salomon Brothers bond index for the United States, Germany and Switzerland. Our data base has 100 term structures starting in January 1990, in monthly steps, for the three countries. Correlation matrices were calculated using the RiskMetrics methodology for estimating volatilities using these historical data. We applied the RiskMetrics methodology with exponential smoothing to the data set in order to estimate volatilities and correlations matrices. We estimate volatilities of the spot interest rates for different maturities, thus we obtain an estimate of the term structure of historical volatilities. (The coefficients of the exponential smoothing model were around 0.5, showing the low persistence of volatility shocks as a result of using monthly data; this is consistent with the properties of GARCH models whereby the magnitude of the correlation coefficients declines exponentially with the time horizon.) We assume that the calculated volatility would remain constant over time and we use this value as the term structure of volatilities in calibrating the binomial lattice. Term structures of interest rates—that are also needed to calibrate the binomial lattices—are available from historical data. All models are built using time steps of one month and the binomial lattice models extend 30-years into the future.

In Figures 2 we show the results of the simulation model for generating scenarios of the 3-month and 6-month forward interest rates for the USD over the period December 1993 to March 1997. As expected the scenarios for the 6-month forecast are more dispersed and in most of the tested periods these scenarios cover the observed interest rates. For the 3-month forecast we have only four possible scenarios and quite often these are not sufficient to cover the observed rate. However, the errors tend to be smaller for the 3-month forecast than for the 6-month forecast. As we will see later these interest rate scenarios seem to be adequate to capture the volatility of the bond indices, especially for the most recent periods. However, the modeling of interest rate scenarios is an area where further improvements of the model are required. For instance we could consider the use of smaller time steps—weekly or daily instead of monthly—and more accurate volatility estimates so that more dispersed scenarios can be generated for the short term rates.

Figures 3 and 4 show the simulated returns of three country indices and of the global index (returns in the base USD currency) over a seven month period from March 1997. (Salomon Brothers index data for October 1997 is missing from our database and this month is not included in the figure.) The 3-month scenario forecasts are shown together with the values realized by the indices. With the exception of the USD index we observe that in all other cases the scenarios are adequate in capturing the true changes of the index. Particularly encouraging is the figure on the global index, where we observe that the scenarios in all cases cover the market values, and the mean value of the scenarios is close to the market value.



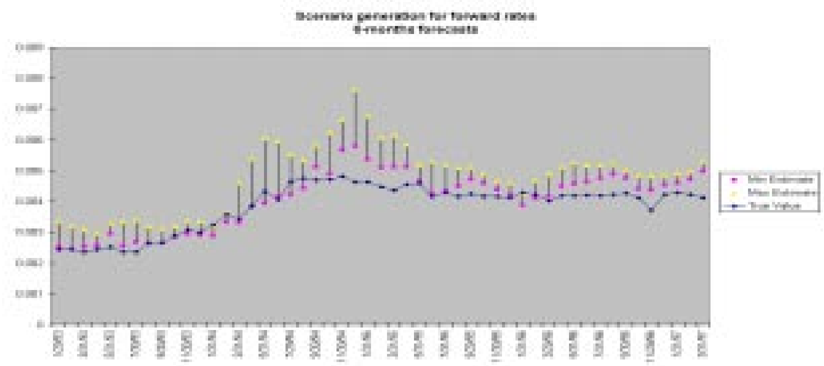
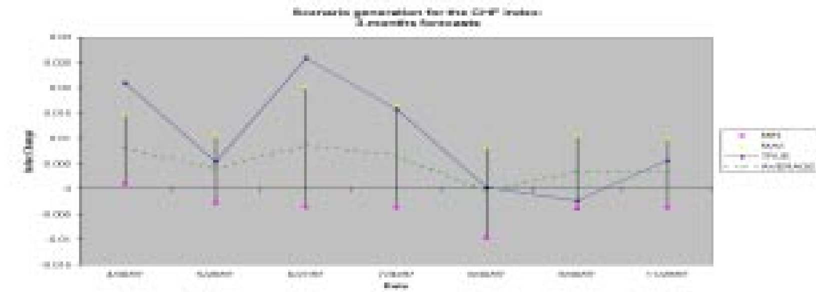
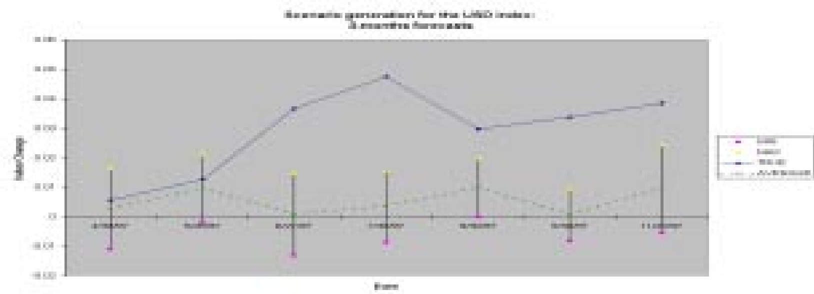


Figure 2: Scenario generation for the forward interest rates in USD. Vertical lines illustrate the range of interest rate scenarios for the date shown and the bullet shows the interest rate that was observed on the particular date. Scenarios were forecast based on information available 3 and 6 months, respectively, before the shown dates.





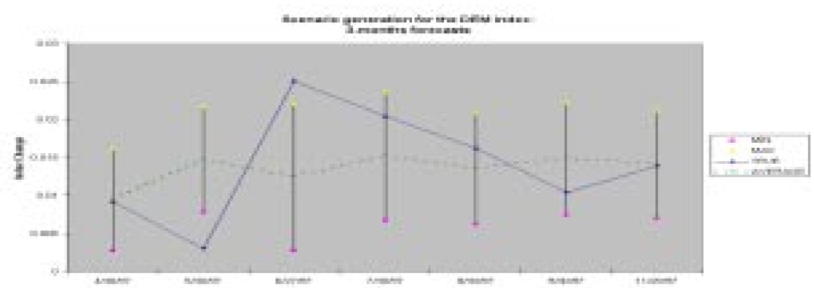


Figure 3: Scenario generation for the Swiss Franc, US Dollar and Deutchmark indices in domestic currency. Scenarios are forecast using information available 3 before the shown dates.

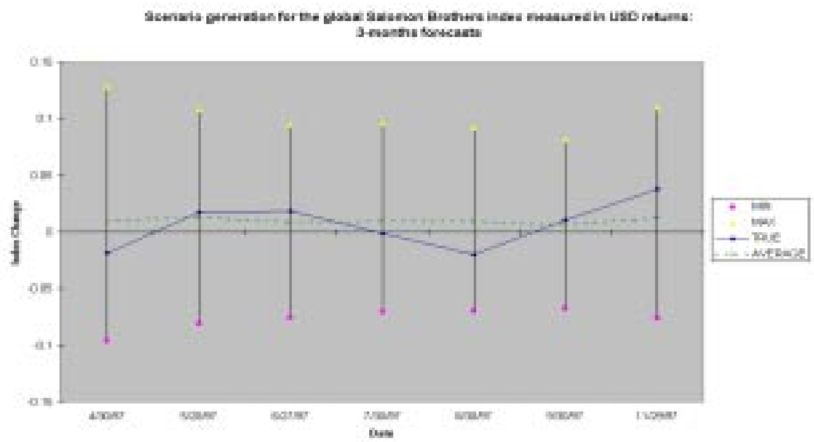


Figure 4: Scenario generation for the Salomon Brothers Global index in the base currency (USD). Scenarios are forecast using information available 3 months before the shown dates.

5 Conclusions

We conclude from the empirical data that further improvements are required in the simulation of interest rates and perhaps better estimates of the volatilities, especially for the USD data. However, the joint generation of returns of the three market indices in the base currency appears to be sufficiently accurate to give some assurance that the stochastic programming models developed in this paper will be effective in tracking the market indices. Obviously the model will fail to track the indices if they are built on scenarios that do not capture potential realizations of the markets. The implementation and testing of the stochastic programming indexation models is the topic of current investigations.

While in the paper we talk only about interest rate and exchange rate risk, there are other sources of risk that are of concern to an international bond portfolio manager. For example, the introduction of EURO in a common European financial market will create the need to model credit spreads between bonds of different maturities of the member countries. These spreads can be calculated as pairwise differentials between the term structures of spot interest rates of the different currencies, much in the same way that Litterman and Iben (1991) model the credit spread of corporate bonds over Treasury bonds. However, modeling the volatility of credit spreads when no historical data is yet available is impossible and we may need to resort to subjective estimates.

Furthermore, it is quite often the case that an enhanced indexation strategy is followed whereby the index can be matched or outperformed by investments in convertible bonds or corporate bonds. Such strategies can readily be modeled in the stochastic programming framework discussed here but we would need to generate scenarios of corporate returns or the returns of convertible bonds. The work of Vassiadou-Zeniou and Zenios (1996) for instance can be used to incorporate corporate bonds in our multicurency bond portfolio model. Nevertheless, the issues discussed here are currently outside the scope of our paper.

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