Integrated Simulation and Optimization Models for Tracking International Fixed Income Indices*

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Abstract

Portfolio managers in the international fixed income markets must address jointly the interest rate risk in each market and the exchange rate volatility across markets. This paper develops integrated simulation and optimization models that address these issues in a common framework. Monte Carlo simulation procedures generate jointly scenarios of interest and exchange rates and, thereby, scenarios of *holding period returns* of the available securities. The portfolio manager's risk tolerance is incorporated either through a utility function or a (modified) mean absolute deviation function. The optimization models prescribe asset allocation weights among the different markets and also resolve bond-picking decisions. Therefore several interrelated decisions are cast in a common framework. Two models—an

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expected utility maximization and a mean absolute deviation minimization—are implemented and tested empirically in tracking a composite index of the international bond markets. Backtesting over the period January 1997 to July 1998 illustrate the efficacy of the optimization models in dealing with uncertainty and tracking effectively the volatile index. Of particular interest is the empirical demostration that the integrative models generate portfolios that dominate the portfolios obtained using classical disintegrated approaches.

1 Introduction

Does international diversification pay? This question has been answered affirmatively since the 1960's when it was first addressed in an article by Grubel (1968). Several authors have shown that international markets are sufficiently uncorrelated so that portfolio risks can be reduced by a well-diversified international portfolio, even when accounting for the currency exchange risk exposure. This observation is true for both the equities and fixed income markets, and the literature on this topic is extensive; see Michaud et al. (1996), Jorion (1989), and the book by Solnik (1996). Today, however, a simplistic affirmative answer to this question is no more acceptable since many interrelated factors need to be considered. For instance, the volatilities of the fixed income markets—specially bond markets—are typically lower than the volatilities of exchange rates, and hence the currency risk of an international portfolio *might* offset any benefits from diversification, see Kaplanis and Schaefer (1991). Even in the equities markets it has been argued recently that multiple acquisitions in a foreign country will not improve significantly the efficiency of a portfolio when accounting for the added currency risk, see Ziobrowski and Ziobrowski (1995).

The globalization of the financial markets only exacerbates the difficulties facing international bond portfolio managers. On the one hand, reductions in capital controls, technological advances in dissemination of information, and increased computing power for tracking portfolios and forecasting market trends led to an explosive growth of international bond trading over the last decade. On the other hand, the very same developments have led to increased synchronization of market returns. Table 1 summarizes the correlation coefficients between U.S. and foreign bond market (domestic) returns for two different time periods. In all cases, except Canada, correlation increased with time thus eroding any benefits from international diversification. However, when we account for currency volatilities, and calculate correlation coefficients with market returns converted to USD we observe that the degree of correlation remained constant or even declined. (Data are obtained from Fabozzi, 1997, ch. 54.) Hence, benefits can still be reaped from diversification, but those are to a large extend due to exchange rate movements. Another important question is now raised: Should currency risk be hedged? The answer is intuitively deceptively simple. If currency risk is hedged then interna-

Market	Domestic returns		USD returns
	1978–1995	1990–1995	1990–1995
Australia	.32	.68	.42
Canada	.71	.51	.49
France	.34	.49	.38
Germany	.50	.51	.33
Japan	.38	.43	.26
Netherlands	.52	.53	.35
Switzerland	.36	.46	.26
U.K.	.39	.49	.39
Intnl. index	.59	.64	.44

Table 1: Correlation coefficients between US and other international bond markets computed using both domestic returns and returns converted to US dollars.

tional market returns are highly correlated and diversification *may* not pay. If currency risk is not hedged then the lowered correlation of market returns may lead to improved portfolio efficiency. To hedge or not to hedge now becomes the question, and the answer is "it depends". Kaplanis and Schaefer (1991) show that under some circumstances it is clearly better to be completely hedged, under other circumstances it is clearly better not to hedge, and in yet other cases partial hedging is the optimal strategy. As readers may suspect the most interesting cases, in the sense of being those circumstances that arise in the financial markets, are those that require partial hedging. The benefits of optimal hedging ratios however are "far from clear" (Kaplanis and Schaefer). Indeed both practitioners (Fabozzi, 1997, chs. 54–55) and academics (Jorion, 1989) agree that hedging should be done on a selective basis when conditions warrant it and the hedging policy may be dependent on the asset mix.

In this setting of complex interactions between market returns and exchange rate volatilities it becomes essential for portfolio managers to have at their disposal a systematic methodology for analyzing the interrelated decisions of (*i*) asset allocation among the international markets, (*ii*) bond-picking in each market, and (*iii*) optimal level of hedging. Mulvey and Zenios (1994) argued that portfolios of bonds should be managed based on the ideas of diversification, looking at the co-movements of the securities under various plausible economic scenarios, instead of using portfolio immunization with simple risk measures such as duration or convexity matching. The ideas of diversification were applied successfully to the management of mortgage securities (Golub et al., 1995), callable bonds (Vassiadou-Zeniou and Zenios, 1996), high-yield bonds (Mulvey and Zenios, 1994) and government bonds (Dupačová and Bertocchi, 1999). This framework is particularly suitable for the management of international bond portfolios when the three interrelated decisions outlined above depend on scenarios of local market returns and exchange rates.

Several optimization models provide the mathematical framework for optimizing the portfolio decisions under the generated scenarios. One class of models stems from the classical mean-variance (Markowitz) optimization models. These models have evolved in several directions to accommodate the investors' utility functions and to handle securities with asymmetric returns. This class consists of single-period models. The most prominent examples of models from this class are expected utility maximization (Ingersoll, 1987) and mean absolute deviation minimization (Sharpe (1971), Konno and Yamazaki (1991), Zenios and Kang (1993)); see, e.g., Zenios (1995) for a classification of these models. More recent research adopts multiperiod optimization models for planning under uncertainty—*stochastic programming*—based on plausible scenarios for the realization of the uncertain data. This class of models dates back to the 1950's and has recently been gaining widespread acceptance for modeling financial decision making problems. See, e.g., Mulvey and Vladimirou (1992), Dembo (1993), Carino, Myers and Ziemba (1998), Carino and Ziemba (1998), Golub et al. (1995), Koskosides and Duarte (1997), Dupačová and Bertocchi (1999), the recent textbooks by Birge and Louveaux (1997) and Censor and Zenios (1997), or the volume by Ziemba and Mulvey (1998).

The adoption of scenario based methodology is particularly suitable in view of recent changes in the global financial markets. For instance, the introduction of the EURO in a unified European financial market raises the issue of credit risk for the member countries. As currently no market data are available to calibrate credit risk volatilities the use of scenarios provides the only viable approach for hedging this type of risk based on plausible scenarios of credit risk premia that can be inferred from economic theory.

In this paper we develop integrated *simulation* and *optimization* models for tracking a composite index of international government bonds. The problem setting, the general modeling approach we take, and two concrete optimization models are described in section 2. Monte Carlo simulation models (section 3) generate joint scenarios of market returns and exchange rates. Extensive empirical results (section 4) illustrate the efficacy of the developed methodology in dealing with the complexities of the international bond markets. The results demonstrate in particular that the integrative models generate portfolios that dominate the portfolios obtained using classical disintegrated approaches.

This paper is the third of a series dealing with the same general problem: Beltratti, Consiglio and Zenios (1999) laid the foundation for this research by developing the general framework of multi-period stochastic programming models for addressing asset allocation, bond picking, and hedging decisions using forward contracts. The hedging model was implemented and tested in a single-period mean-variance formulation of the asset allocation problem in Beltratti, Laurent and Zenios (1998). The present paper develops and tests empirically a single-period formulation of the joint problem of asset allocation and bond-picking. In order to fully validate the general framework of Beltratti, Consiglio and Zenios we need to incorporate hedging decisions in the current paper and also to extend it to the multiperiod formulation. The positive empirical results in Beltratti, Laurent and Zenios and in the current paper provide evidence that these further developments are likely to produce even more powerful modeling tools.

2 The optimization models

2.1 Problem definition

Indexed fixed-income funds have gained in popularity over the last decade for reasons that are well documented in the literature, see, e.g., Fabozzi (1997, ch. 47), and we adopt this particular management philosophy for developing the models. We consider in particular the problem of a portfolio manager whose mandate is to track an international index of fixed income securities. This problem was defined in Beltratti, Consiglio and Zenios (1999) and this section restates the earlier formulation.

A bond index in each market $i = 1, 2, \ldots, m$, is constructed by creating a representative sample Φ_i of size n from the universe of eligible bonds Ω_i (we assume for simplicity of notation that all samples are of equal size). For each security $j = 1, 2, \ldots, n$, in the representative set, the index specifies its relative weight ϕ_j^i which reflects the capitalization structure of the universe set Ω_i with bonds that have characteristics identical or similar to the jth bond. The global index is represented by the relative weights γ_i assigned to the country indices based on the market value of each index.

In practice the sets Ω_i may consist of thousands or even hundreds of thousands of bonds (differing by issuer, issue data, maturity date, coupon payments etc), while the representative sets Φ_i consist of hundred or so representative securities. Global bond indices, such as the Salomon Brothers Global index, consist of holdings in the bond markets of the major industrialized nations (i.e., $m = 7$ for the G7 index).

The manager of an international bond portfolio must determine the fraction of the portfolio value invested in each of the m markets, and to pick specific bonds from each market Ω_i to add to the portfolio. These decisions are typically made in two steps. An asset allocation committee determines first the exposure of the portfolio to each market, and then traders identify mispriced bonds in each market and construct the country-specific portfolio. This portfolio has to track the country specific index as well, so constraints on duration, maturity etc may be imposed on the trader. Finally, once the country-specific funds are constructed the currency exposure may be hedged.

We denote the portfolio weights—in proportion of total wealth—allocated to the index of each country $i = 1, 2, \ldots, m$, by w_i, and the proportional holdings in bond $j = 1, 2, \ldots, n$, in each country by x_{ij}. An asset allocation model determines optimal weights w_i^*, and a bond picking model then specifies optimal holdings such that $\sum_{j=1}^{n} x_{ij}^* = w_i^*$. An integrated model determines jointly the optimal holdings x_{ij}^* and weights w_i^*. We illustrate the modeling process in Figure 1.

One of the key contributions of this paper is in developing models for integrating the

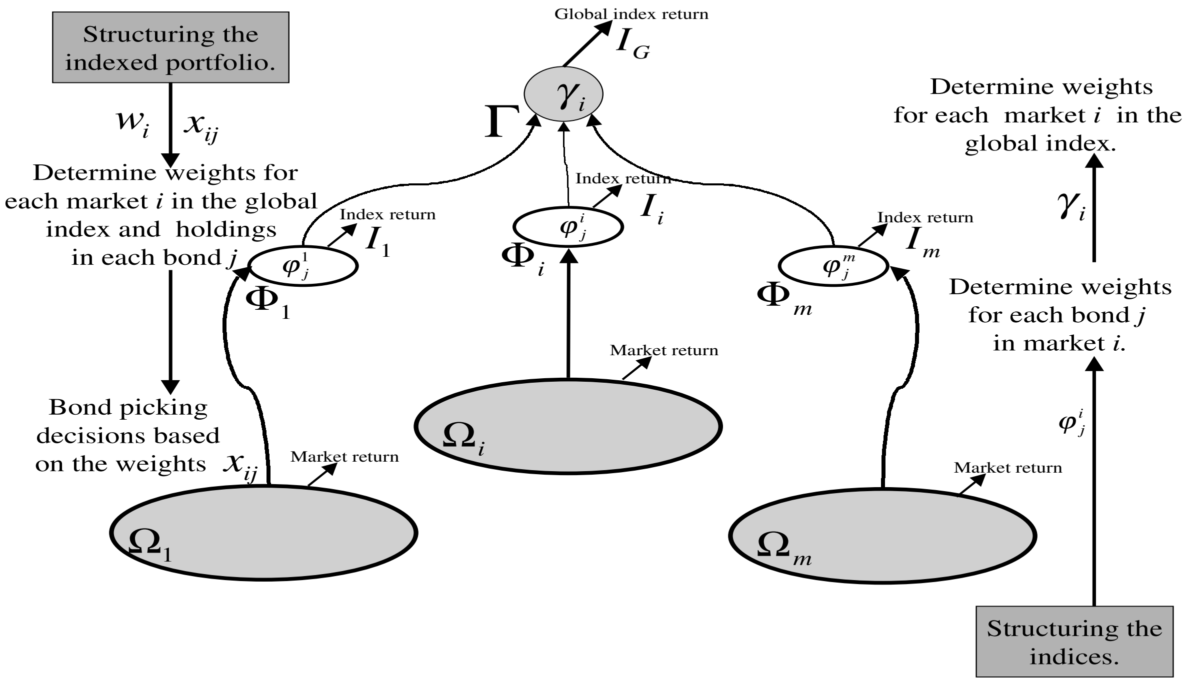


Figure 1: The process for structuring the market indices and the indexed portfolio.

financial decisions of asset allocation and bond-picking in a common framework. The significance of integrating several interrelated financial decisions in a common framework finds widespread acceptance in the finance literature, see, e.g., Merton (1990), and Holmer and Zenios (1995). However, empirical studies substantiating the benefits of integration are scant. A recent paper in this direction integrating stock and bond decisions in a common framework with encouraging results is due to Konno and Li (1998).

Having defined the portfolio manager's problem we now specify concrete optimization models for addressing it. We give first a model that determines asset allocation and bond-picking decisions in an integrated fashion. Then we give the two models that determine first the asset allocation and then the bond-picking decisions.

2.2 The integrated indexation models

We introduce first an index set $\{1, 2, \ldots, S\}$ for *scenarios* of the uncertain market data; we assume that scenario s occurs with probability π_s. For each realized scenario s from this set we have an observation of the index return for the ith market, I_i^s. The return of the international composite index is given in the base currency by $I^s = \sum_{i=1}^m e_i^s \gamma_i I_i^s$, where e_i^s is the exchange rate appreciation (or depreciation) of the ith currency against the base currency under scenario s. The exchange rate appreciation (or depreciation) is computed as the ratio of the exchange rate under scenario s (E_i^s) to the current exchange rate (E_0^s), $e_i^s = E_i^s / E_i^0$. We use r_{ij}^s to denote 1 plus the rate of return of the jth bond in the ith currency under scenario s. The return of the portfolio in the base currency is given by $R_p^s = \sum_{i=1}^m e_i^s \sum_{j=1}^n r_{ij}^s x_{ij}^s$. The mean value of the random variable bond returns r_{ij} is denoted by $\bar{r}_{ij}$, and of the random variable exchange rate appreciation e_i by $\bar{e}_i$.

With this notation we can now define a mean absolute deviation model for minimizing deviations of the portfolio return from the composite index. The mean absolute deviation of the tracking error between the random variable index return I and the random variable portfolio return R_p is given by

$$w(R_p) \doteq \mathcal{E}\left(\mid (R_p - I) - \mathcal{E}(R_p - I) \mid \right), \tag{1}$$

where $\mathcal{E}()$ denotes expectation of its random variable argument. Since R_p and I take discrete values from the sets $\{R_p^s \mid s = 1, 2, \ldots, S\}$ and $\{I^s \mid s = 1, 2, \ldots, S\}$, respectively, and the means of these random variables are equal by construction of the indexed portfolio, an unbiased estimate for $w(R_p)$ is obtained from

$$\hat{w}(R_p) = \sum_{s=1}^{S} \pi_s \mid R_p^s - I^s \mid . \tag{2}$$

The mean absolute deviation (MAD) model is now written as:

$$\begin{aligned}
&\underset{x}{\text{Minimize}} \quad \hat{w}(R_p) && (3)\\
&\text{s.t.} \quad \sum_{i=1}^{m} \bar{e}_i \sum_{j=1}^{n} \bar{r}_{ij} x_{ij} \geq \rho, && (4)\\
&\qquad \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = 1, && (5)\\
&\qquad x_{ij} \geq 0, \qquad \text{for all } i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. && (6)
\end{aligned}$$

Here ρ is a user-specified parameter denoting the target expected portfolio return. The model can be reformulated as a linear program that maximizes expected return while it differentially penalizes downside from upside tracking error. (See Zenios and Kang (1993) for details on the reformulation.) In particular, an infinite penalty can be imposed for downside risk and no penalty for upside risk. However, downside risk can not be totally eliminated and a margin of negative error ϵ is introduced which is set equal to the smallest possible value for which a feasible solution can be obtained. The modified model—which we call the *downside* mean absolute deviation model—maximizes the expected portfolio return while it restricts the downside error to be within ϵ. It is written as:

$$\underset{x}{\text{Maximize}} \quad \sum_{s=1}^{S} \pi_s \sum_{i=1}^{m} e_i^s \sum_{j=1}^{n} r_{ij}^s x_{ij} \tag{7}$$

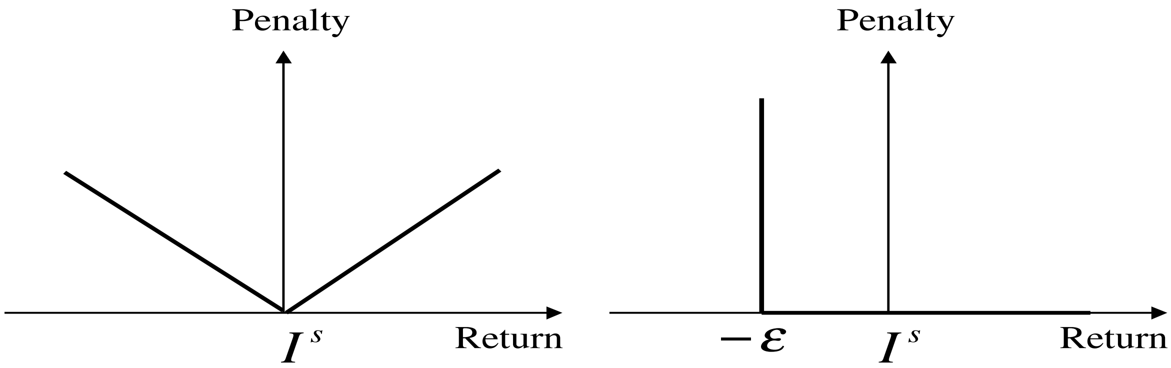


Figure 2: Symmetric and asymmetric penalty of the deviation of portfolio return from the index return. The function on the left is the penalty function minimized by the mean absolute deviation model; the function on the right is the penalty minimized by the downside mean absolute deviation model.

$$\text{s.t.} \qquad \sum_{i=1}^{m} e_i^s \sum_{j=1}^{n} r_{ij}^s x_{ij} \geq I^s - \epsilon, \qquad \text{for all } s = 1, 2, \ldots, m, \tag{8}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = 1, \tag{9}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{10}$$

Figure 2 illustrates the symmetric penalty function implied by the mean absolute deviation model and the asymmetric penalty function implied by the downside MAD model.

We note that portfolios constructed by the downside MAD model deviate from the widely held notion that a tracking portfolio is one that mimicks an index both in upside and downside movements. We are not arguing in favour of our model againist mimicking portfolios—this is a managerial policy decision—and we point out that the philosophy of mimicking portfolios can be modeled by using the symmetric penalty function. In the rest of the paper we test the model that limits only downside risk.

A closely related model uses a utility function of excess return of the portfolio over the index in order to penalize tracking error. Define R_e as the random variable excess return of portfolio return over the index with scenario values $R_e^s = R_p^s / I^s$, and let $U(R_e)$ denote the utility of excess return. The expected utility maximization model is written as:

$$\underset{x}{\text{Maximize}} \quad \sum_{s=1}^{S} \pi_s U(R_e^s) \tag{11}$$

$$\text{s.t.} \qquad R_e^s = \frac{\sum_{i=1}^{m} e_i^s \sum_{j=1}^{n} r_{ij}^s x_{ij}^s}{I^s}, \tag{12}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = 1, \tag{13}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \tag{14}$$

In our work we use the Morgenstern-von Neumann isoelastic utility function $U(R_e^s) = (R_e^s)^\lambda/\lambda$ where λ parametrizes the decision makers' risk aversion. A popular choice is to set $\lambda = 0$ which yields the optimal growth strategy with the utility function $U(R_e^s) = \log R_e^s$. However, with this choice of objective function the denominator in the definition of R_e^s, i.e., the index return, is a constant in the optimization model, and the model selects portfolios that are growth optimal without consideration of any deviations from the underlying index.

2.2.1 Operational model

The models of this section can be extended to account for the cost of transactions in rebalancing a portfolio, and to allow for cash infusion or withdrawal, liquidity and diversification constraints and similar operational constraints. In order to incorporate some of these technical considerations the model variables need to be expressed not in percentages of total wealth, as was done above, but in face value. We give here the detailed formulation of a model that incorporates operational constraints as well. We define first the model variables in terms of face value, and define additional parameters.

$X_{ij} =$ face value purchased of security j in the ith currency,

$Y_{ij} =$ face value sold of security j in the ith currency,

$Z_{ij} =$ face value of security j in the ith currency that remains in the indexed portfolio,

$v =$ amount invested in cash in the base currency.

Now define various constants and model parameters:

$b_{ij}^0 =$ face value holdings in the starting portfolio of security j in the ith currency,

$c_0 =$ initial endowment in cash in the base currency,

$\zeta_{ij}^0 =$ selling price of security j in the ith currency at the start of the planning horizon,

$\delta =$ transaction costs.

The initial value of the portfolio is now given by

$$V_0 = c_0 + \sum_{i=1}^{m} E_i^0 \sum_{j=1}^{n} \zeta_{ij}^0 b_{ij}^0.$$

The value of the portfolio at the end of the holding period under scenario $s = 1, 2, \ldots, S$, is given by

$$V^s = \sum_{i=1}^{m} E_i^s \sum_{j=1}^{n} r_{ij}^s \zeta_{ij}^0 Z_{ij} + r_0^s v$$

where r_0^s is the riskless rate of return of the base currency, and Z_{ij} is given by

$$Z_{ij} = b_{ij}^0 + X_{ij} - Y_{ij}, \text{ for all } i = 1, 2, \ldots, m, \text{ and } j = 1, 2, \ldots, n.$$

The return of the portfolio is then given by $R_p^s = V^s / V_0$ and the downside mean absolute deviation model can be written as follows:

$$\underset{X,Y,Z}{\text{Maximize}} \quad \sum_{s=1}^{S} \pi_s R_p^s \tag{15}$$

$$\text{s.t.} \quad Z_{ij} - X_{ij} + Y_{ij} = b_{ij}^0, \tag{16}$$

$$c_0 + \sum_{i=1}^{m} E_i^0 \sum_{j=1}^{n} \zeta_{ij}^0 Y_{ij} = \sum_{i=1}^{m} E_i^0 \sum_{j=1}^{n} X_{ij}(\zeta_{ij}^0 + \delta) + v \tag{17}$$

$$R_p^s \geq I^s - \epsilon \tag{18}$$

$$X_{ij}, Y_{ij}, Z_{ij} \geq 0, \tag{19}$$

Equations (16) are inventory balance constraints for all securities j in all currencies i. Equation (17) is the cashflow balance constraint. Inequality (18) is the penalty function of the downside mean absolute deviation model and holds for all scenarios $s = 1, 2, \ldots, S$. The bounds (19) restrict all variables to be nonnegative so that short sales are not allowed. To this model we can also add constraints of the form

$$\zeta_{ij}^0 Z_{ij} \leq L \left(\sum_{i=1}^{m} E_i^0 \sum_{j=1}^{n} \zeta_{ij}^0 Z_{ij} \right)$$

to restrict the holdings in the jth security in the ith currency below a fraction L of the total portfolio value.

2.3 The non-integrated models

We develop now two models that address separately the asset allocation and the bond picking problems. These models mimic more closely the operations of international bond portfolio managers that usually determine first their currency exposure, and then determine the portfolio holdings in each currency. We formulate the mean absolute deviation models only, and by analogy with section 2.2 we can formulate modified downside MAD models or expected utility maximization models. Similarly following section 2.2.1 we can develop disintegrated models with operational constraints. We do not give all these formulations here.

2.3.1 Asset allocation model

The random variable return R_p of a portfolio with exposure w_i to the ith currency take scenario values

$$R_p^s = \sum_{i=1}^{m} e_i^s I_i^s w_i.$$

The following mean absolute deviation model

$$\begin{aligned}
&\underset{w}{\text{Minimize}} && \hat{w}(R_p) && (20)\\
&\text{s.t.} && \sum_{i=1}^{m} w_i = 1, && (21)\\
& && w_i \geq 0, \qquad \text{for all } i = 1, 2, \ldots, m. && (22)
\end{aligned}$$

determines the optimal currency exposure $(w_i^*)_{i=1}^m$.

2.3.2 Bond-picking model

Having obtained the optimal currency weights $(w_i^*)_{i=1}^m$ we can now state the model for determining the portfolio composition in each currency. The random variable return of the portfolio in the ith currency R_{pi} takes scenario values

$$R_{pi}^s = e_i^s \sum_{j=1}^{n} r_{ij}^s x_{ij}.$$

The following mean absolute deviation models solve the bond picking problem:

For each $i = 1, 2, \ldots, m$, solve:

$$\begin{aligned}
&\underset{x}{\text{Minimize}} && \hat{w}(R_{pi}) && (23)\\
&\text{s.t.} && \sum_{j=1}^{n} x_{ij} = w_i^*, && (24)\\
& && x_{ij} \geq 0, \qquad \text{for all } i = 1, 2, \ldots, m, \;\; j = 1, 2, \ldots, n. && (25)
\end{aligned}$$

These models determine the optimal weights $(x_{ij}^*)_{j=1}^n$ for bonds holdings in each currency $i = 1, 2, \ldots, m$.

3 Monte Carlo scenario generation models

In order to implement the portfolio optimization programs we need a simulation procedure to generate interest rate and exchange rate scenarios. Once such scenarios are generated the calculation of bond prices—in both domestic and base currency—conditioned on the observed interest rates is straightforward; see, e.g., Mulvey and Zenios (1994). The model provides a structural interpretation of bond returns. Returns are not simply forecasted on the basis of information variables; instead they are deduced from the scenarios of interest rates.

Models for jointly estimating interest and exchange rates are prevalent in the finance literature. However, the problem of jointly modeling these processes in a way that is consistent with market observations is a difficult one and the success of various models in accurately predicting interest and exchange rate changes has been limited. Meese and Rogoff (1983) compared the predictive abilities of a variety of exchange rate models. Their key result was that no existing structural exchange rate model could reliably out-predict the naive alternative of a random walk at short- and medium-run horizons, even when aided by actual future values of the regressors. This extremely negative finding has never been entirely convincingly over-turned despite many attempts, see Frankel and Rose (1995). The simple random walk model of the exchange rate has become the standard benchmark for empirical exchange rate performance, no matter how uninteresting it is per se.

We adopt in this paper a Monte Carlo simulation procedure that encompasses two methodologies based on recent finance literature that are widely accepted by practitioners. Interest rates are generated using binomial lattice models of the term structure, see, e.g., Black, Derman and Toy (1990). Joint scenarios of interest and exchange rates are generated using the Value-at-Risk methodology, see RiskMetrics (1996, ch. 7).

3.1 Interest rate scenario generation

Interest rates are generated from a binomial lattice. The short-term rates for time period t are assumed to be lognormally distributed, and the t-period spot rate implied from the short-term rates is assumed to have a mean value equal to the t-period spot rate given by the current term structure of interest rates, and volatility equal to the t-period volatility of interest rates. Assuming further a discretized binomial approximation one obtains a model such as the one given by Black, Derman and Toy (1990) whereby the short-term rate at period t and at the σth state of the binomial lattice is given by:

$$r_t^\sigma = r_t^0 (k_t)^\sigma, \tag{26}$$

where $\{r_t^0\}_{t=0}^T$ and $\{k_t\}_{t=0}^T$ are parameters of the binomial lattice estimated using the procedures described by Black, Derman and Toy (1990), and all states of the lattice are equiprobable; the lattice describes a risk-neutral world. A Black-Derman-Toy lattice is calibrated for each currency separately using the term structure of interest rates and the term structure of volatility for each country.

3.2 Exchange rate scenario generation

Interest rates in the two countries are key determinants of the exchange rate between the currencies. Hence, we adopt an approach whereby exchange rate scenarios are conditional on the interest rates of the two currencies, the base currency and the foreign currency. We assume that the logarithms of the ratios of exchange rates at period t to period $t-1$, and the logarithms of the ratios of spot interest rates at period t to period $t-1$ follow a multivariate normal distribution. This is a standing assumption in RiskMetrics and the Value-at-Risk methodology, and it is also well justified in the context of our models[1].

We denote the logarithms of the ratios of all random variables by the p-dimensional random vector ω, where $p = mT + (m-1)$. The dimension of ω is equal to the number of currencies times the number of time periods—for the spot rate random variables—plus number of currencies minus 1—for the exchange rates of each currency against the base currency. The real probability density function of ω is given by

$$f(\omega) = (2\pi)^{-p/2} \mid \Sigma \mid^{-1/2} \exp\left[-\frac{1}{2}(\omega - \mu)'\Sigma^{-1}(\omega - \mu)\right], \tag{27}$$

where μ is the expected value of ω and Σ is the covariance matrix.

The covariance matrix Σ and the expected values μ of all random variables involved in the model are available daily through RiskMetrics and can be used to build the multinormal distribution. Once the multivariate normal distribution is built we can use it in Monte Carlo simulations, using either the standard Cholesky factorization approach (see, e.g., RiskMetrics, 1996, ch. 7) or the scenario generation procedures based on principal component analysis discussed in Jamshidian and Zhu (1997). We do not use any one of these approaches in our simulations, since we wish to incorporate the additional information on the interest rate random variables from the binomial lattice.

We proceed as follows: The multivariate normal variable ω is partitioned into two subvectors ω_1 and ω_2, where ω_1 is the vector of dimension $p_1 = mT$ of random variables corresponding to interest rates, and ω_2 is the vector of dimension $p_2 = m-1$ of exchange rates. The expected value vector and covariance matrix are partitioned similarly as

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}. \tag{28}$$

It is known (see, e.g., Jobson, 1992, ch. 7) that the marginal probability density function of ω_2 given $\omega_1 = \omega_1^*$ is

$$f(\omega_2 \mid \omega_1 = \omega_1^*) = (2\pi)^{-p_2/2} \mid \Sigma_{22.1} \mid^{-1/2} \exp\left[-\frac{1}{2}(\omega_2 - \mu_{2.1})'\Sigma_{22.1}^{-1}(\omega_2 - \mu_{2.1})\right], \tag{29}$$

[1]Daily and weekly rates do not follow normal distributions; however, there is lack of empirical evidence against normality for monthly data such as those used in our model.

where the conditional expected value and covariance matrix are given by

$$\mu_{2.1}(\omega_1^*) = (\mu_2 - \Sigma_{21}\Sigma_{11}^{-1}\mu_1) + \Sigma_{21}\Sigma_{11}^{-1}\omega_1^*, \tag{30}$$

and

$$\Sigma_{22.1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}, \tag{31}$$

respectively. Exchange rate scenarios for period t conditioned on value of the interest rates vector given by ω_1^* can be generated from today's exchange rates and the multivariate normal variables from (29) through the expression

$$e_{t_i} = e_{0_i} e^{\sigma_i \sqrt{t}\omega_{2_i}},$$

where e_{0_i} is today's exchange rate for the i currency, σ_i is the 1-period volatility of the ith exchange rate, and ω_{2_i} is the conditional multivariate random variable corresponding to the ith exchange rate.

Figure 3 illustrates the conditional probabilities for several exchange rate scenarios. On the same figure we plot the exchange rate that was realized *ex post* on the date for which the scenarios were estimated. Note that the same exchange rate value may be obtained for various different scenarios of interest rates and samples drawn from (29); the figure plots several points with the same exchange rate value but different conditional probabilities. Figure 4 is a discretized approximation (histogram) of the probability density function of the exchange rates.

3.3 Empirical testing of the simulation models

The models described above allow us to generate a large number of scenarios of interest and exchange rates. Scenarios are sampled from these models using the procedure described in Beltratti, Consiglio and Zenios (1999).

We backtested the simulation models on a set of data obtained from the Salomon Brothers bond index for the United States, Germany and Switzerland. Our data base has 100 term structures starting in January 1990, in monthly steps, for the three countries. Volatilities and correlation matrices were calculated from these historical data using the RiskMetrics methodology with exponential smoothing. We estimate volatilities of the spot interest rates for different maturities, thus estimating the term structure of historical volatilities. (The coefficients of the exponential smoothing model were around 0.5, showing the low persistence of volatility shocks as a result of using monthly data; this is consistent with the properties of GARCH models whereby the magnitude of the correlation coefficients declines exponentially with the time horizon.) We assume that the calculated volatility would remain constant over time and we use this value as the term structure of volatilities in calibrating the binomial lattice. All models are built using time steps of one month and the binomial lattice models extend 30-years into the future.

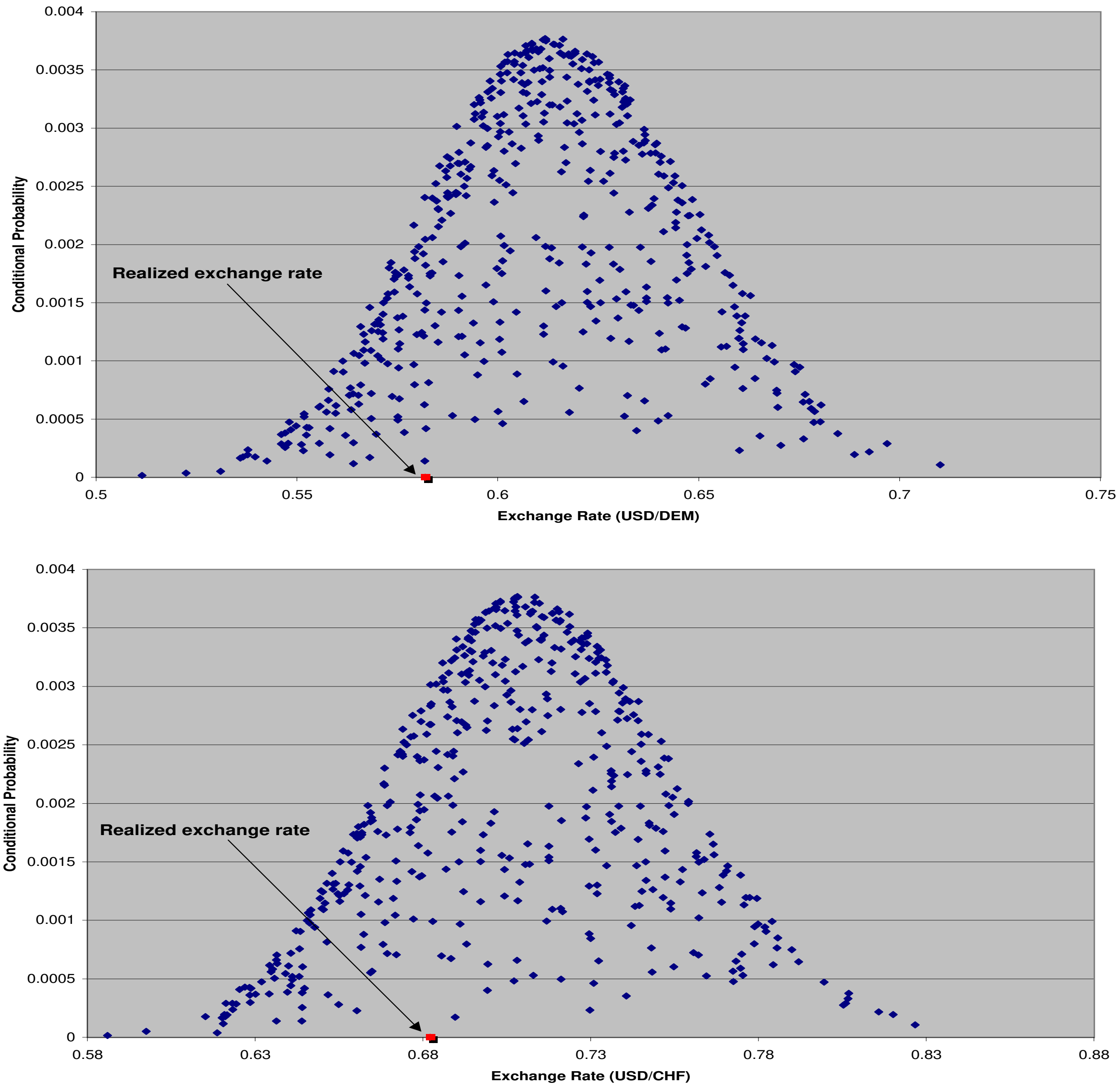
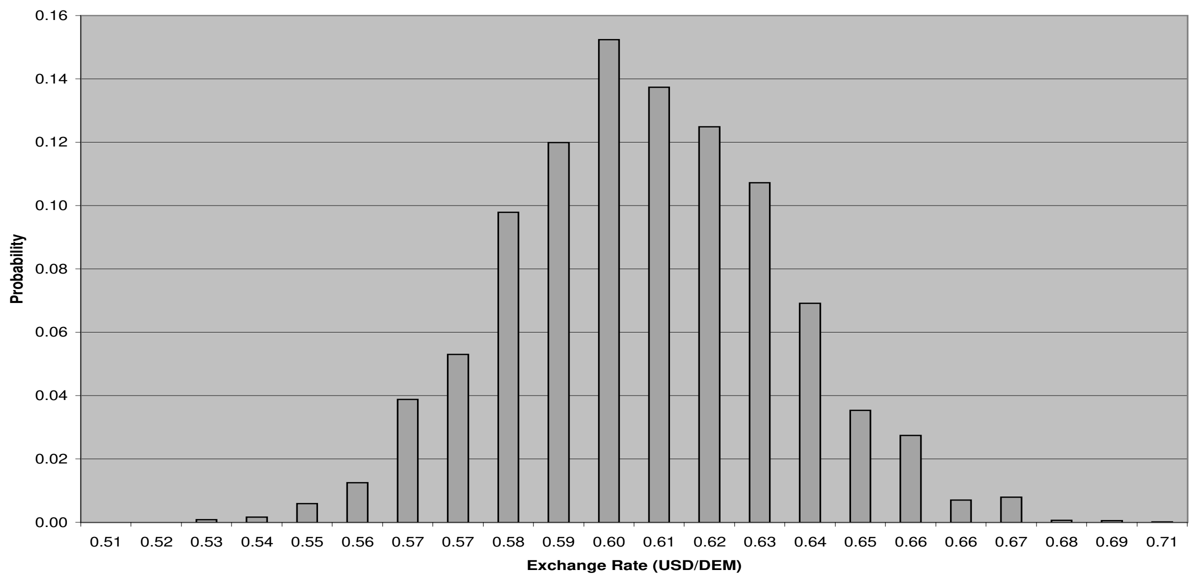


Figure 3: Exchange rate scenarios and their conditional probabilities for the DEM and CHF against the USD.



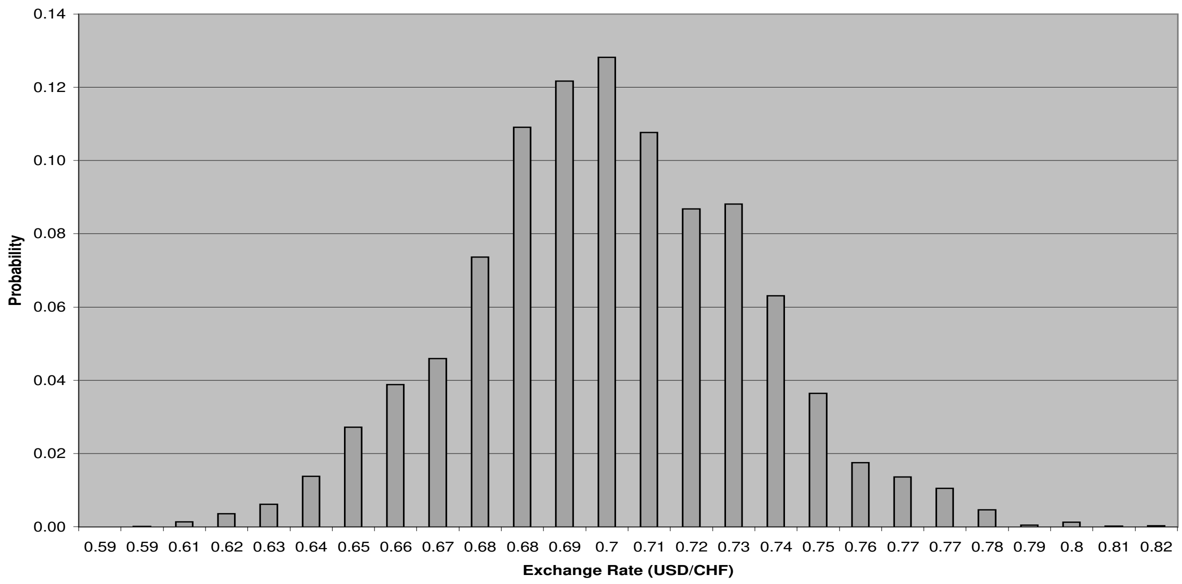


Figure 4: Histogram of the exchange rate scenarios.

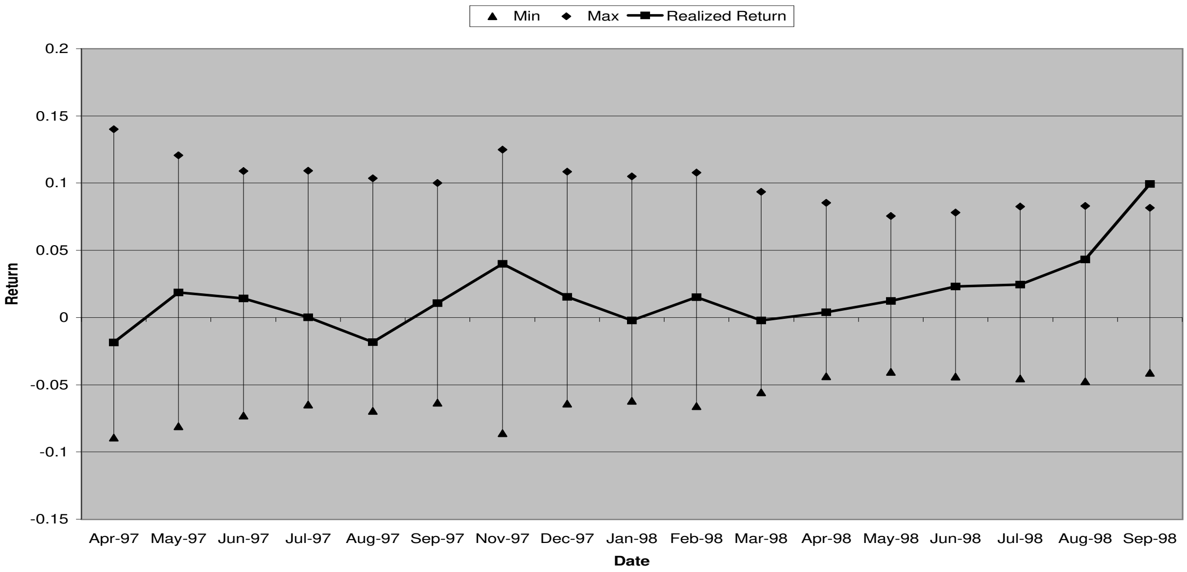


Figure 5: Scenario generation for the composite international index in the base currency (USD). Scenarios are forecast using information available three months before the shown dates.

Figures 5 shows the simulated returns of the composite index (returns in the base USD currency) over an eighteen month period from March 1997. (Salomon Brothers index data for October 1997 is missing from our database and this month is not included in the figure.) The 3-month scenario forecasts are shown together with the values realized by the index. It is worth noting that the period of backtesting (mid 1997 to late 1998) was particularly stressful for the international financial markets. During this period we witnessed the devaluation of currencies in Korea and South Africa, the easing of interest rates in the United States by the Fed, the Russian crisis. It is remarkable that the models perform as well as they did without relying on human foresight for the impending crises. If such foresights are available they can be incorporated in the scenario-based models (Koskosides and Duarte, 1997).

4 Empirical results: model validation

We now turn our attention to the validation of the indexation models, using *ex post* analysis of the performance of portfolios developed from the models. In general we expect that an "optimal" portfolio will perform well under the scenarios that were input in the modeling process. ¿From the users' perspective however we are interested in studying the performance of the portfolios obtained by the models using simulated scenarios, under the realized market returns.

Our testing proceeds along the following general lines. Using term structure and exchange rate data from January 1990 to December 1996 we estimate volatilities and correlations. Starting then on January 1997 and using the term structure of that date and the historical volatilities and correlations we calibrate the Monte Carlo simulation models, generate scenarios of index returns and security returns, and run the optimization models to select a tracking portfolio. We then move a month forward (February 1997) at which point we know the precise index return, security returns, and exchange rates and can thus determine the performance of our tracking portfolio and the tracking error. Thus we complete one step of the backtest. We then use the term structure of February 1997 to update the volatility and correlation estimates, recalibrate the Monte Carlo simulation models and repeat the exercise. The process is repeated until July 1998, which is the last month for which we have data. In the sections that follow we report results obtained with the application of the above methodology.

4.1 Tracking a composite index of the international bond markets

Using the Salomon Brothers G7 index we created a composite index consisting of equal holdings in two major currencies (USD and DEM) and the currency of the sponsoring institution (CHF). We backtested both the downside MAD and the expected utility maximization models. Figure 6 shows the asset growth of 1000 USD invested in January 1997 in the index and in portfolios generated using the models. We observe that both models perform well in tracking the volatile index. Calculating the Sharpe ratio of the portfolios we obtain -0.068 (index), 0.369 (MAD) and 0.443 (utility maximization); these are very encouraging statistics for the model performance. Nevertheless, Figure 6 shows the results of a single realization of the random index returns: the historical realization. In some sense this is the only realization that matters. But how would the portfolio perform under alternative realizations of the random returns? Figure 7 illustrates the tracking error of a typical portfolio under simulated scenarios. We observe from these simulated data that the tracking error is small, and is limited to a lag of 1% (annualized) below the index under worst case scenarios, while the lead over the index is 2.2% under the most favorable scenario.

4.1.1 Limiting the currency exposure

It is typically the case in international portfolio management that an explicit limit on currency exposure is imposed no matter what the predictions may be for specific markets based on historical observations. For instance, if we were to rely on our historical data for the estimation of correlations and volatilities we would conclude that the DEM index would dominate both the USD and CHF index, and an optimizer would allocate 100% of the portfolio assets to DEM bonds. This is not a prudent allocation, especially for a USD

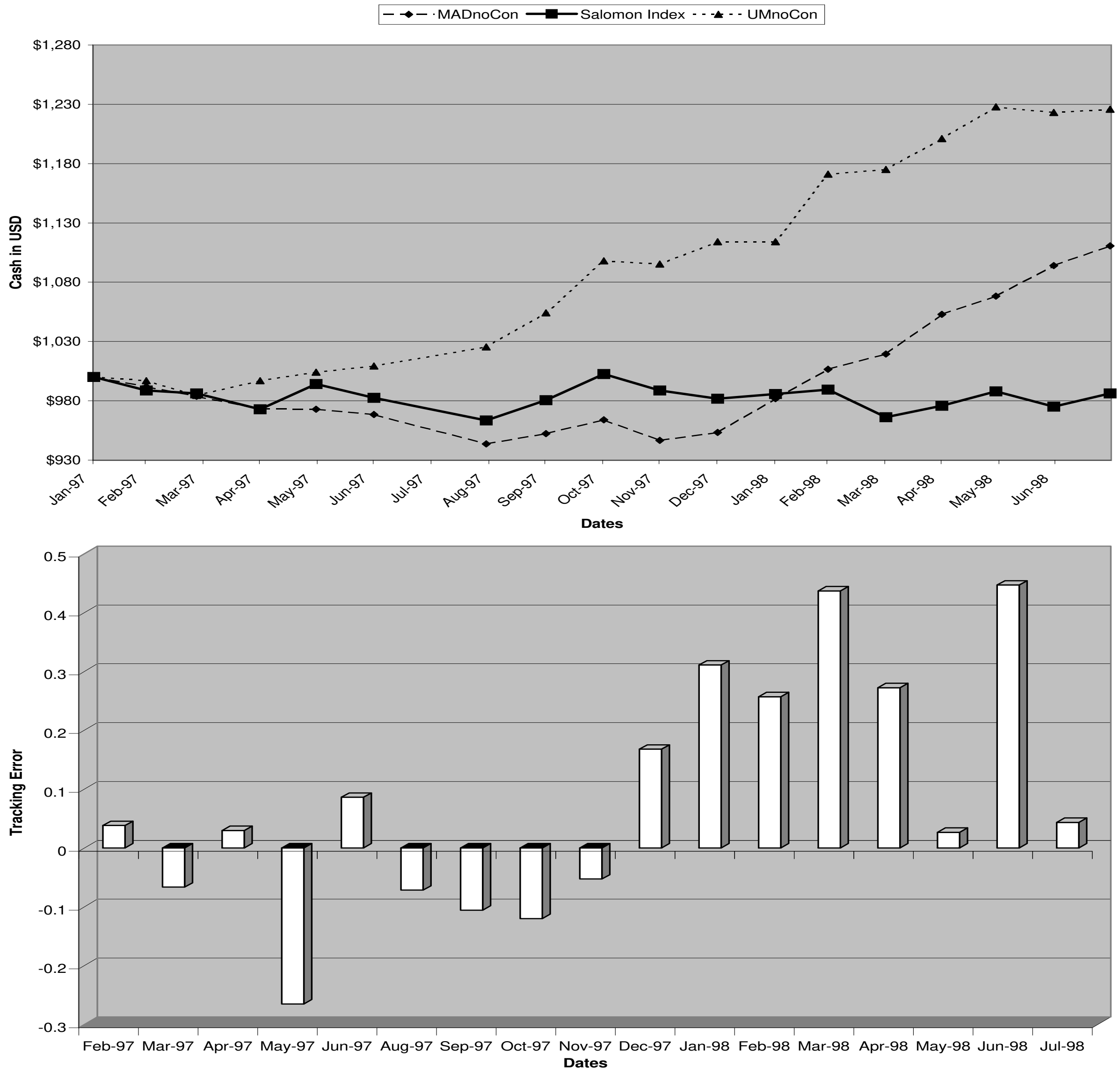


Figure 6: Value of a 1000USD investment in the composite index and in indexed portfolios constructed using the downside mean absolute deviation and the expected utility maximization models. (The figure at the bottom shows the monthly (annualized) tracking error of the downside MAD portfolio.)

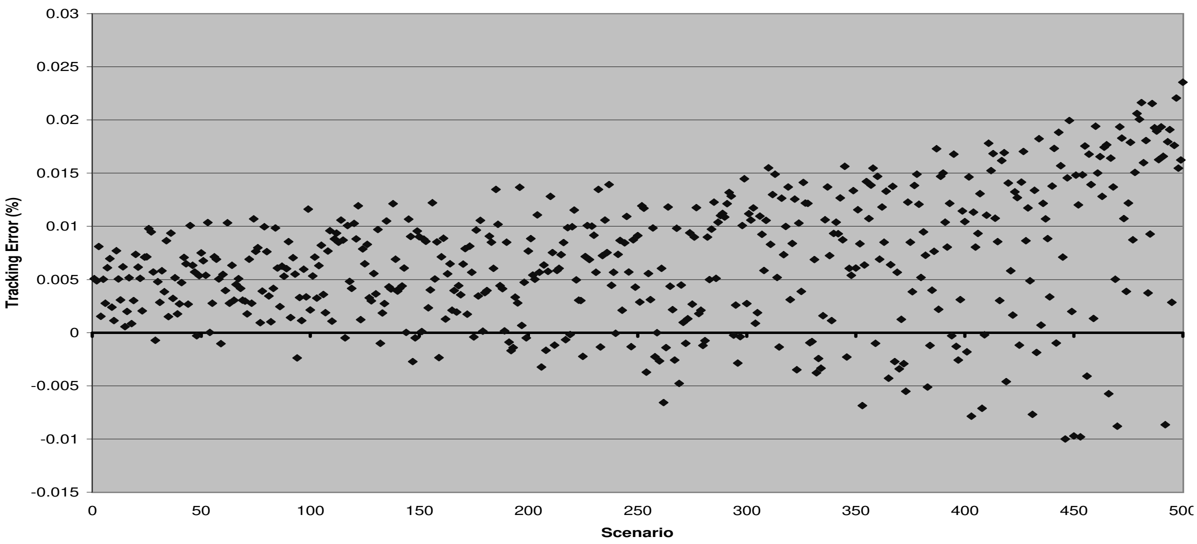


Figure 7: Tracking error in annualized basis points of a typical portfolio against the composite index for 500 simulated scenarios of returns and exchange rates.

denominated investor. Hence, we run the model by constraining the exposure in each currency to remain within ±15% of the composition of the index. Figure 8 shows the asset growth of 1000USD invested in January 1997 in the index, in a portfolio generated using the MAD model without any limit on the currency exposure, and in a portfolio that stays within the 15% band. The Sharpe ratio of the unconstrained portfolio is 0.369 and of the constrained portfolio is 0.411. These results show that *ex post* the limits on the currency exposure are well justified. A model optimizer should not be used without some judicious guidance by practical considerations.

4.1.2 Incorporating operational constraints

We proceed to investigate the effect of imposing additional operational constraints to the indexed portfolios. First, we limit the holdings in any particular bond not to exceed 5, 10 or 20% of the total portfolio value. The growth of 1000USD invested in the MAD indexed portfolios, with these holdings bounds, are shown in Figure 9. The Sharpe ratios are 0.199 (for the 5% bound on holdings), 0.208 (10%) and 0.202 (20%). Note that these are *ex post* results so we do not necessarily expect the least constrained model to reach the best Sharpe ratio. We observe instead that the tightly constrained portfolio (at the 5% bound) lags in performance to the less constrained portfolios, but beyond the 10% bound there is no performance improvement. In any event, the model performance is uniformly good if we consider that the Sharpe ratio of the index is -0.068.

Finally we impose the additional constraint that no holdings in CHF should exceed 150, due to liquidity requirements in the Swiss market. We also keep the bound of

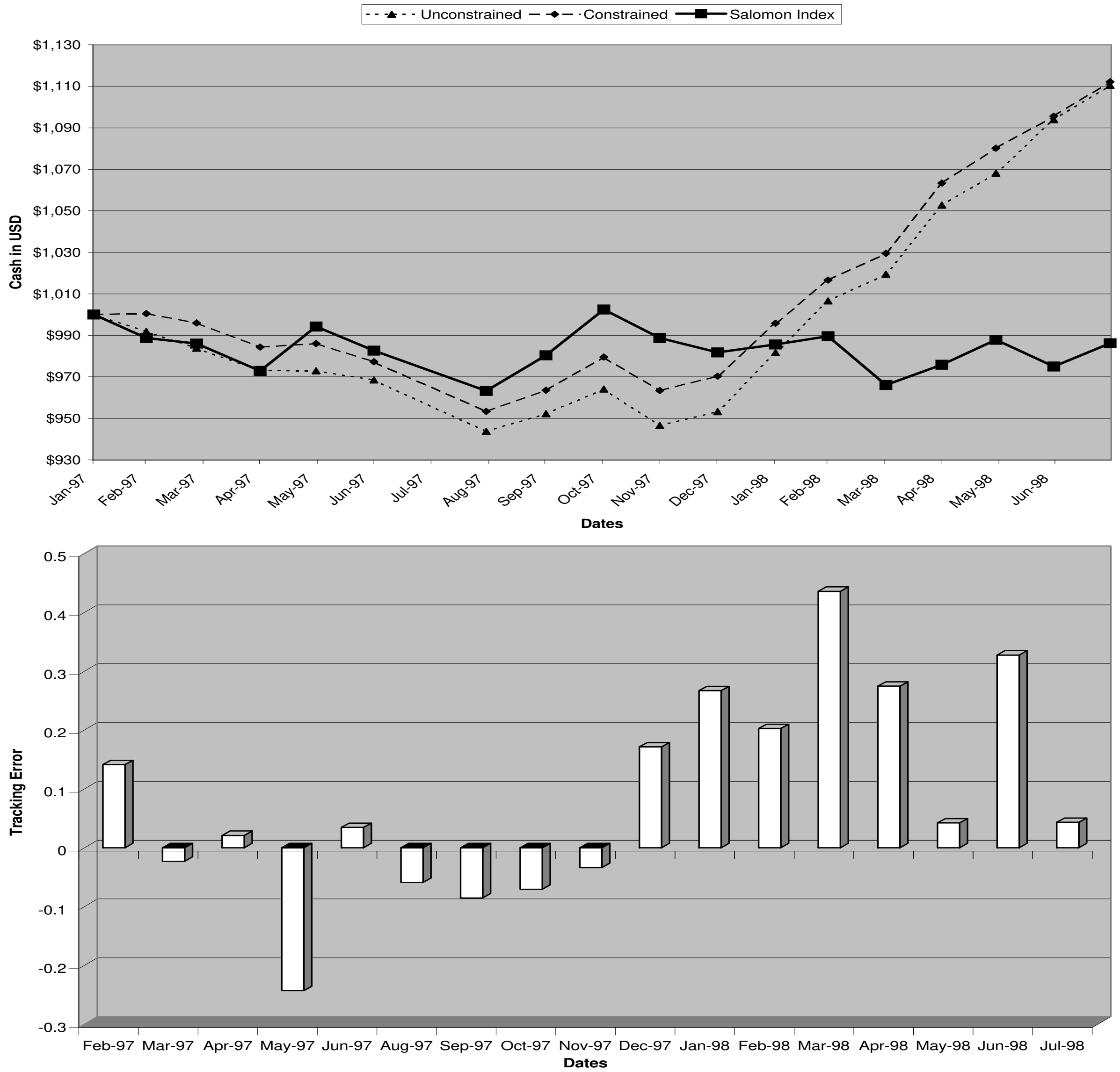


Figure 8: Value of a 1000USD investment in the composite index, in an indexed portfolio without limits on the currency exposure, and in an indexed portfolio with exposure to each currency within $\pm 15\%$ of the corresponding index exposure. (The figure at the bottom shows the monthly (annualized) tracking error of the portfolio with currency exposure limits.)

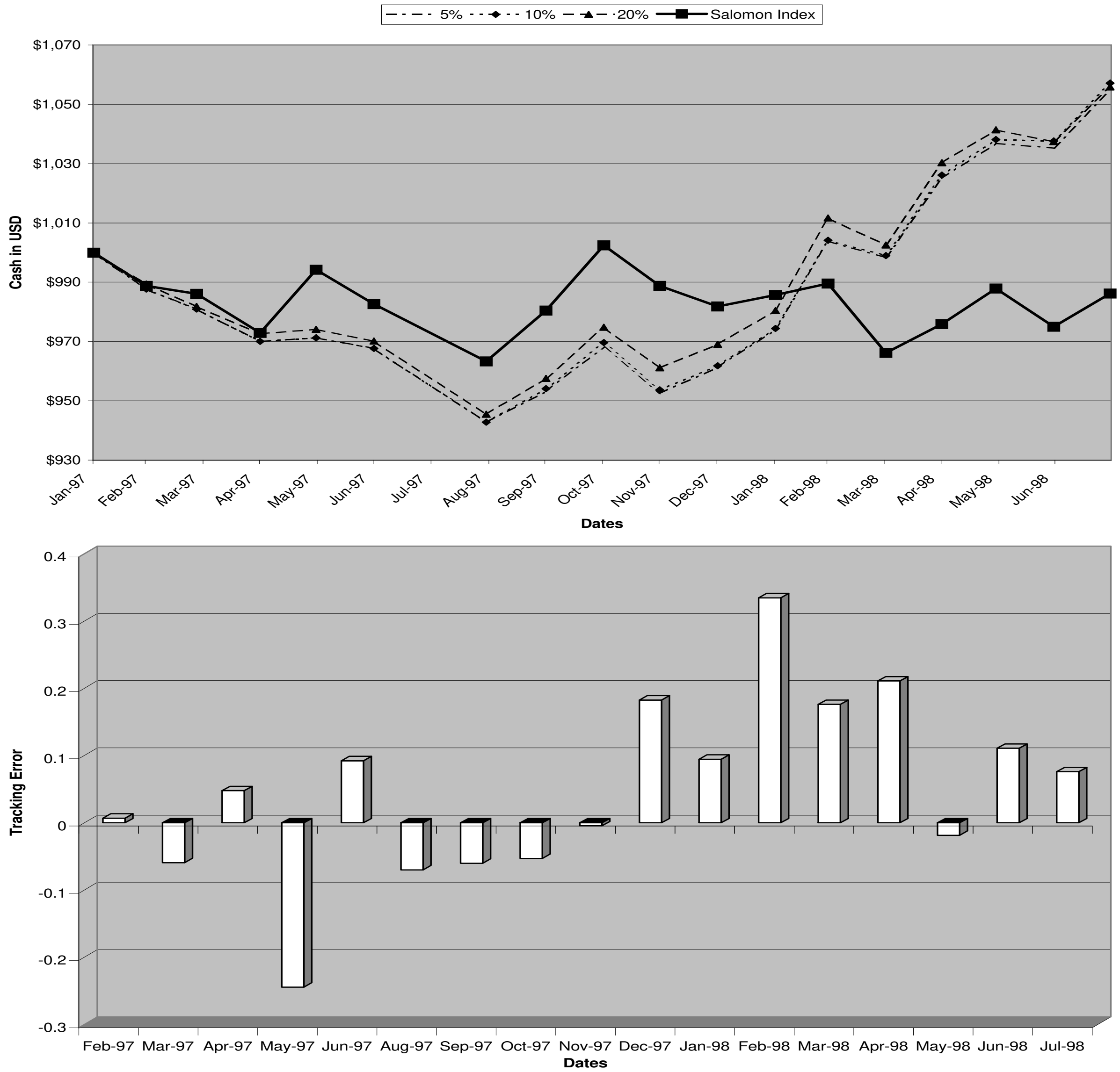


Figure 9: Value of a 1000USD investment in the composite index, and in indexed portfolios with bond holdings limited to 5%, 10%, and 20%, respectively, of the total portfolio value. (The figure at the bottom shows the monthly (annualized) tracking error of the portfolios with 20% limit on bond holdings.)

10% in the holdings of any bond and restrict the currency exposure to remain within the ±15% band around the composition of the index. This set of constraints reflects essentially all operational considerations of the portfolio managers we interracted with. Results are shown in Figure 10, and the Sharpe ratio for the operational portfolios is 0.192 (a small drop to 0.189 is realized when the currency exposure is limited). This figure summarizes the most encouraging results of our testing since it demonstrates that we can build indexed portfolios that satisfy our operational requirements and appear to outperform the composite index with out-of-sample data.

4.2 Duration matched portfolios

For the operational indexed portfolios we calculate their duration for each month during the backtesting period, and contrast it to the duration of the index. The results are shown in Figure 11. We observe that the portfolios have durations similar, but *not* identical, to the index duration. The fact that the portfolio durations are close to the index duration allow portfolio managers—accustomed to the idea of duration matching their portfolios against the index— to gain confidence in the scenario based model. However, duration is only a first-order approximation to the interest rate changes. The scenario based models deviate from the duration matching concept when the volatility of the generated scenarios is not adequately represented by duration measures. Figure 12 summarizes the exposure of the indexed portfolios to the three currencies. We note that the exposure is similar but not identical to the index exposure, while occasionally the portfolios invest in cash. Figure 13 illustrates the compositions of two typical portfolios. We note that while some holdings are at the limit (10%) imposed by the user, several holdings are determined based on their expected return and correlations with other securities. This is an indicator of a well-specified model that does not simply identity mispriced securities or data imperfections and then return the maximum holding in these securities allowed by the user-specified bounds. If this were the case—as is sometimes the case in portfolio "optimization" models—then we would have built a sophisticated machinery for performing simple pricing calculations. The empirical results of this section show that the scenario optimization models capture the intricate interactions of the international fixed income markets, even under several operational constraints.

4.3 Random portfolios

When sophisticated mathematical models are used it is customary to question whether a more naive approach could perform equally well. Recall, for example, the experiment by the editors of *Forbes* magazine who chose a portfolio of stocks in 1967 by throwing darts at the stock-market page of the New York Times. A decade later this portfolio had beaten the broad market indices, and its returns were exceeded by very few professional money managers.

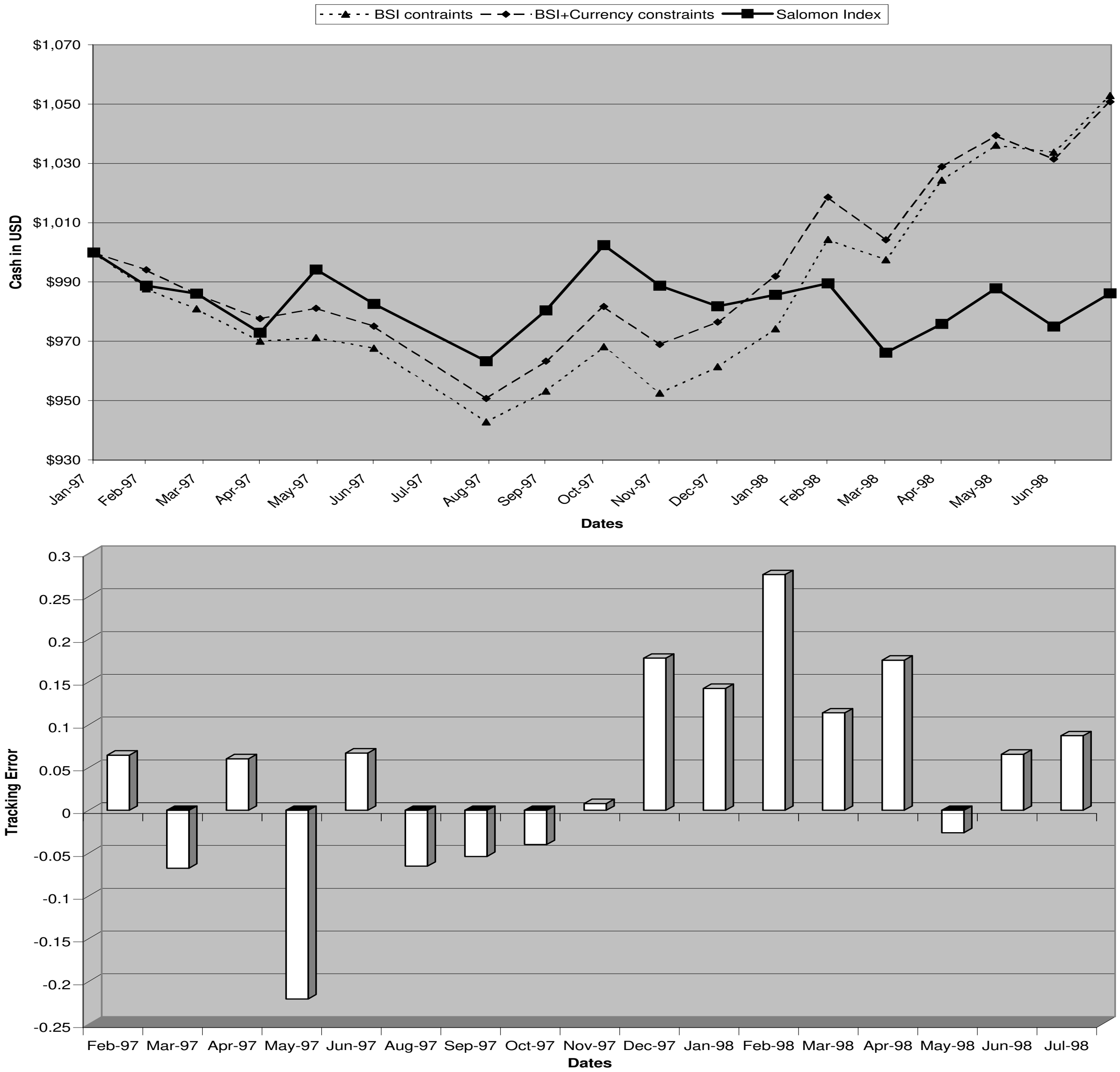


Figure 10: Value of a 1000USD investment in the composite index, and in indexed portfolios constrained by operational restrictions on liquidity (marked: BSI constraints) and currency exposure(marked: BSI+currency constraints). (The figure at the bottom shows the monthly (annualized) tracking error of the portfolios with liquidity and currency constraints.)

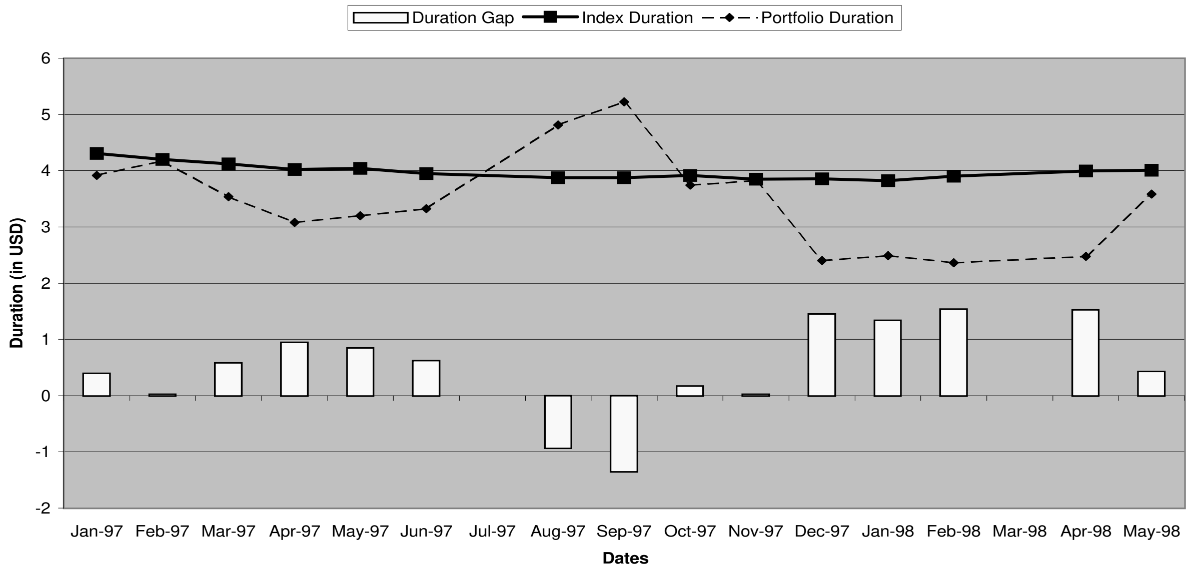


Figure 11: Duration gap between the indexed portfolios and the composite index over the testing period.

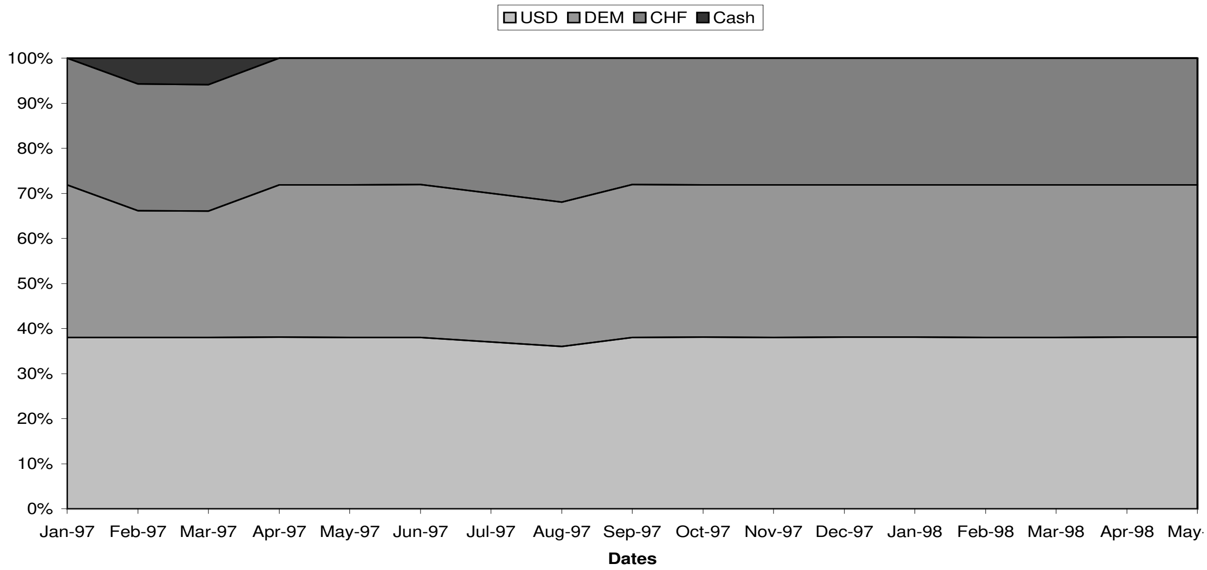
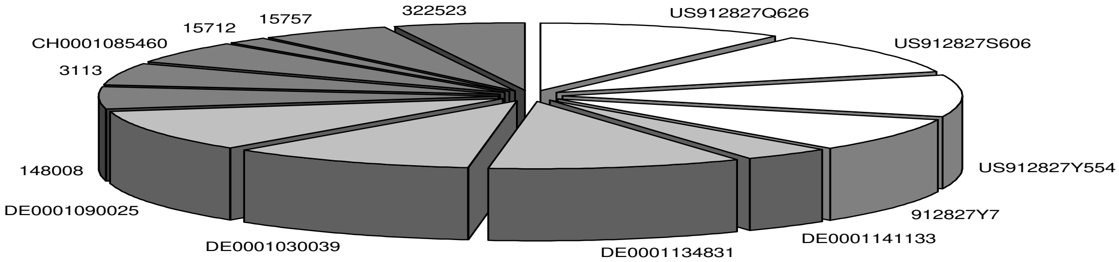
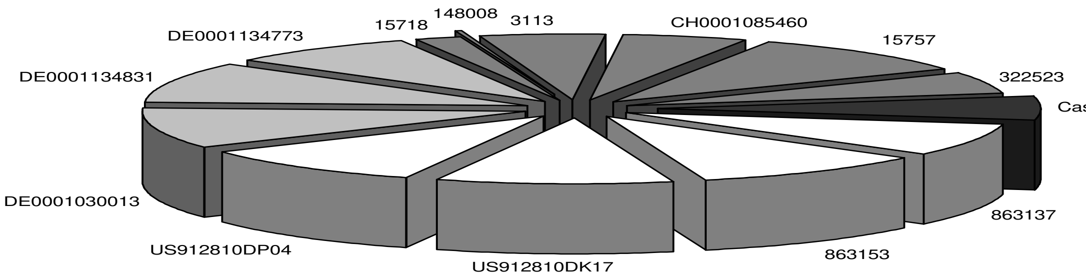


Figure 12: Exposure of the indexed portfolios to the three currencies and in cash over the testing period.



Currency	Coupon	Maturity	Duration	Holdings
USD_90	6.88%	31/07/99	2.34	10.0%
USD_98	7.75%	31/01/00	2.73	10.0%
USD_127	7.00%	15/07/06	7.10	10.0%
USD_128	6.63%	31/07/01	3.97	8.0%
DEM_1	7.00%	13/01/00	2.77	3.8%
DEM_22	9.00%	22/01/01	3.54	10.0%
DEM_52	8.00%	21/01/02	4.33	10.0%
DEM_79	7.13%	29/01/03	5.12	10.0%
CHF_10	4.25%	06/01/14	12.34	5.3%
CHF_18	6.25%	07/01/03	5.19	5.3%
CHF_20	4.25%	08/01/08	8.98	5.3%
CHF_21	4.25%	02/12/11	11.05	1.7%
CHF_25	6.50%	05/02/02	4.22	5.3%
CHF_26	5.50%	06/01/05	6.71	5.3%



Currency	Coupon	Maturity	Duration	Holdings
USD_14	13.38%	15/08/01	3,61	8.0%
USD_17	14.25%	15/02/02	3,89	10.0%
USD_27	13.75%	15/08/04	5,32	10.0%
USD_31	11.25%	15/02/15	9,59	10.0%
DEM_18	8.50%	20/02/01	3,57	10.0%
DEM_23	9.00%	22/01/01	3,47	9.9%
DEM_44	7.75%	21/02/00	2,77	8.2%
CHF_9	4.25%	25/02/12	11,22	2.0%
CHF_10	4.25%	06/01/14	12,36	0.2%
CHF_18	6.25%	07/01/03	5,12	5.3%
CHF_20	4.25%	08/01/08	8,95	5.3%
CHF_25	6.50%	05/02/02	4,42	10.0%
CHF_26	5.50%	06/01/05	6,65	5.3%
USD				5.7%

Figure 13: Composition of two illustrative portfolios.

We tested the performance of randomly selected portfolios of international government bonds over the period of our backtesting. The average monthly returns, standard deviation of returns, and average downside errors over this period are shown in Figure 14 for 100 randomly selected portfolio. The same information is also reported for the Salomon Brothers index and for several portfolio constructed with our models. We observe that the model-selected portfolios dominate. It is interesting though to note that only (approximately) 25% of the randomly selected portfolios are dominated by the index. A manager who throws darts at the bond pages is likely to perform well against the broad market index but not againist competitors who use sophisticated models.

4.4 The efficacy of the integrated indexation models

We have argued in the theoretical part of this paper in favor of integrating several interrelated financial decisions in a common framework. The previous subsections with the empirical experiments have shown that the models perform well *ex post*, but it is not yet clear whether the integration pays. We develop now efficient frontiers—with the MAD model—using both the integrated and the disintegrated models. For the disintegrated models we first solve the asset allocation model and set the currency exposure to the one that yields the highest expected return. We then develop the efficient frontier of the portfolios in the three currencies such that the total exposure in each currency is equal to that determined by the asset allocation model. We note that the indexation models pick the point on the efficient frontier with the highest expected return. Figure 15 shows the efficient frontiers obtained with both the integrated and the disintegrated models, and we note that the integration entails substantial benefits for the portfolio managers, by reducing substantially the volatility of the portfolio returns while also increasing somewhat the expected return.

A careful inspection of the portfolio allocations, however, reveals that the integrated model generates poorly diversified portfolios. Left without any operational constraints the model would occasionally generate portfolios with holdings up to 99% in bonds in a single currency. We repeated the comparison between the integrated and disintegrated models, imposing bounds on the total exposure in a single instrument. The results are summarized in Figure 16. We observe—in comparison to Figure 15—that the efficiency gap of the integrated model over the disintegrated one is somewhat reduced. The integrated model with bounds still yields dominating portfolios, and from Figure 16 we also observe that the portfolios of the integrated model, with bounds, are now well diversified, similarly to the portfolios of the disintegrated models.

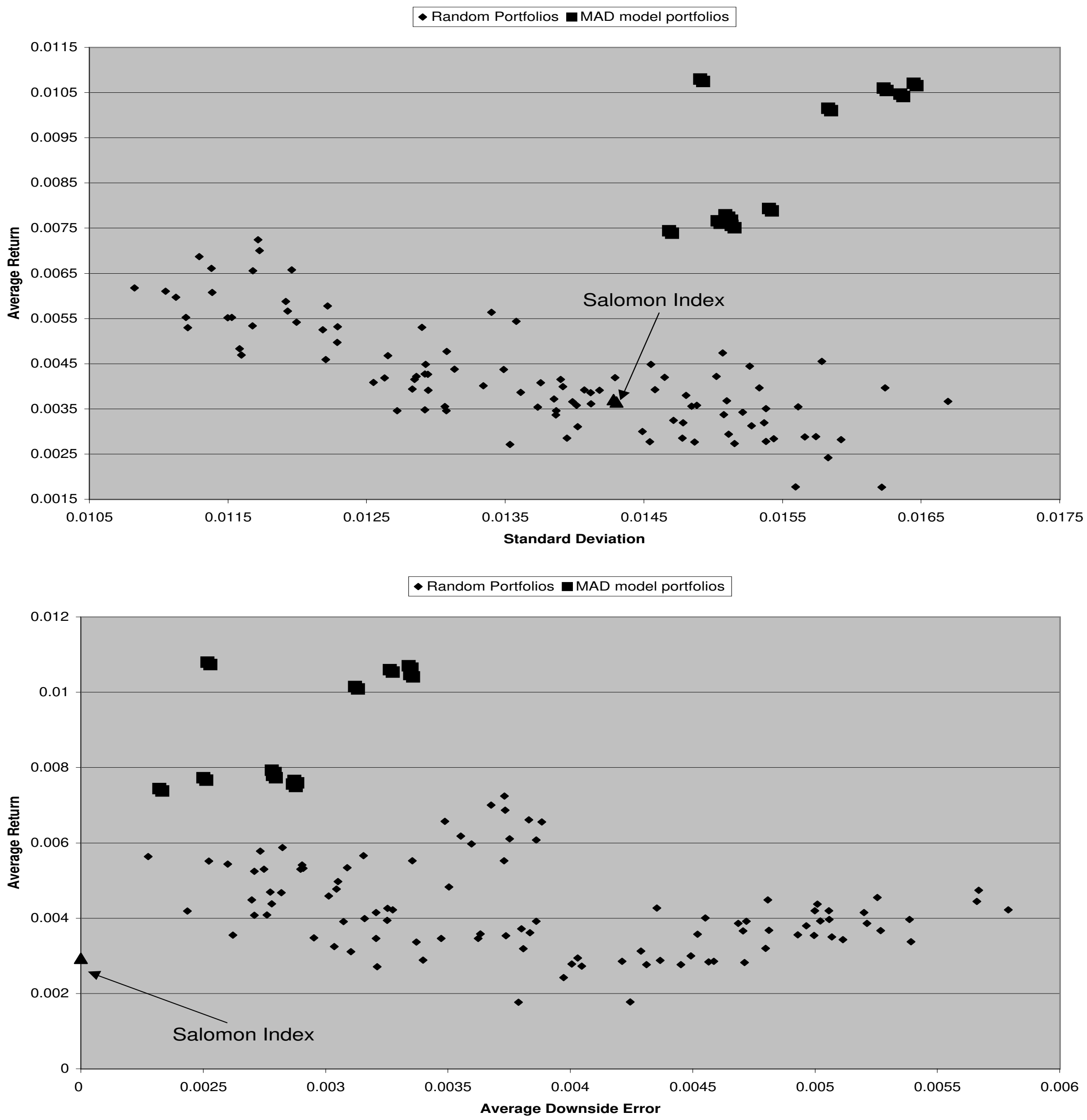


Figure 14: Performance of several model portfolios and 100 random portfolios during the testing period Jan. 1997 to June 1998. Top figure: Average monthly returns and variance of returns for the randomly selected portfolios, the market index, and the model portfolios. Bottom figure: Average month returns and average downside errors for the randomly selected portfolios, the market index, and the model portfolios.

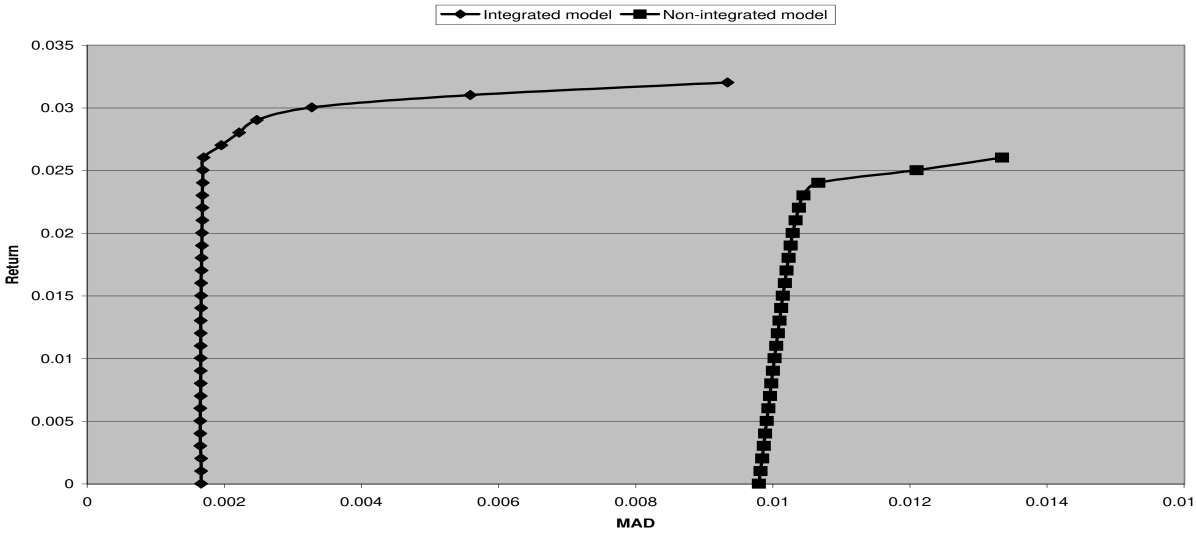


Figure 15: The positive effects of integrating several financial decisions in a common framework: the efficient frontier generated with the integrated model dominates the frontier generated using the disintegrated models.

5 Conclusions

State-of-the-art models from scenario optimization and Monte Carlo financial simulations are viable tools for managing the complexities of the international bond markets. The specific models developed and tested in this paper were shown to be very effective in solving a particular problem from international fund management, namely the tracking of a composite government bond index. We believe that similar results can be achieved in other areas of international fixed income portfolio management, and also in the management of both equities and fixed income portfolios. The developed models allow us to integrate in a common framework several interrelated decisions, and empirical investigations have demonstrated that such integration yields substantial benefits to the portfolio managers. The integration efforts should continue in the direction of incorporating hedging decisions and also in addressing other forms of risk, such as credit risk. This paper and its two predecessors provide the foundation for developing these additional models, while the empirical results provide evidence that further developments are likely to succeed in addressing these practical problems in a meaningful way.

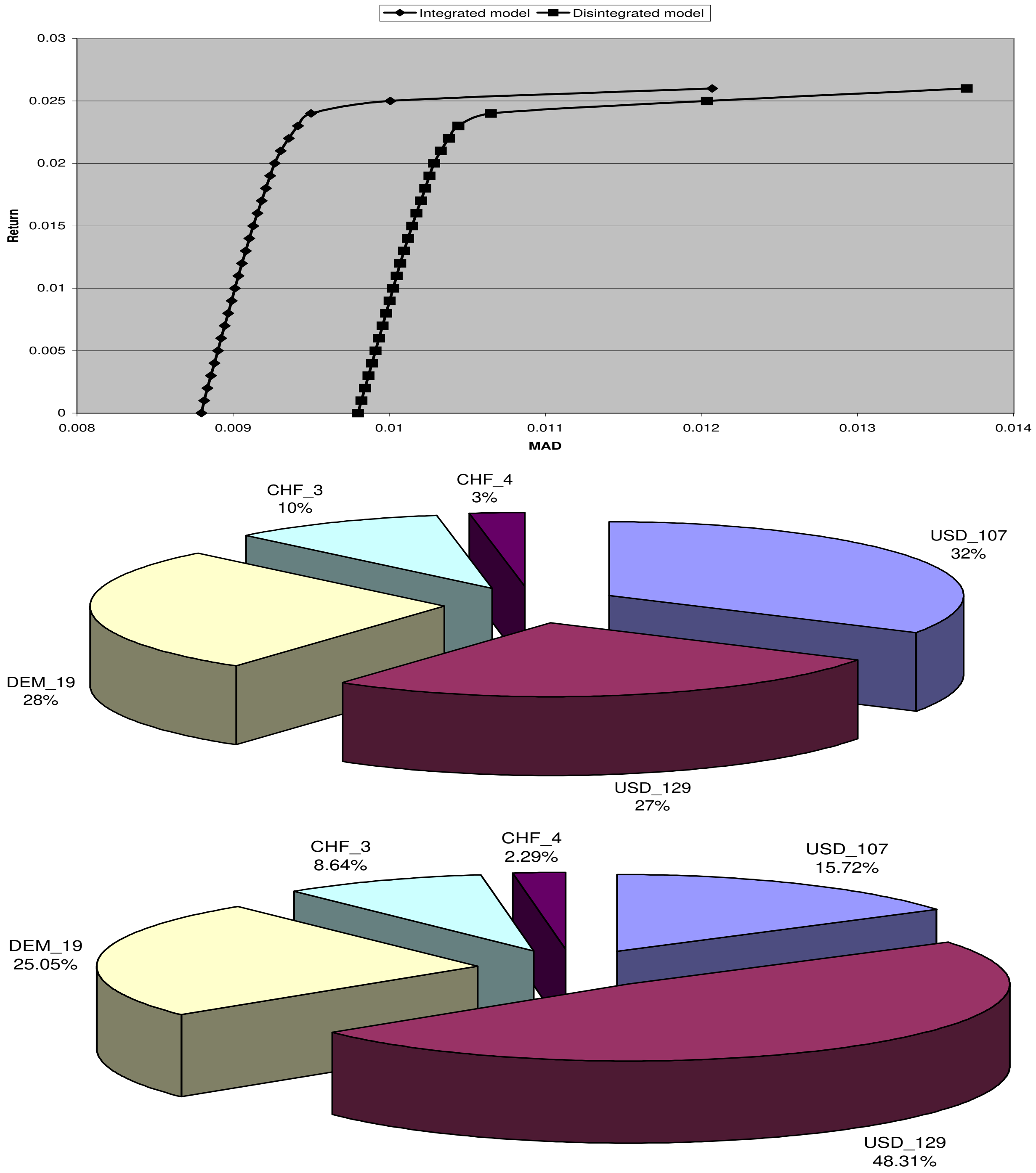


Figure 16: The effects of integrating several financial decisions in a common framework with the addition of bounds on holdings in a single instrument: Top figure: the efficient frontier generated with the integrated and with the disintegrated models. Bottom figures: composition of portfolios generated with the integrated and with the disintegrated models.

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