An analysis of Italian financial data using extreme value theory

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Abstract

The purpose of the paper is to show some methods of extreme value theory through an analysis of Italian financial data. We mainly focus on the Peaks over Thresholds (POT) method and we apply it to the daily logarithmic returns of the FIAT share from 1986 to 1999. We get a satisfactory modelling of the distribution, under the i.i.d. assumption, especially in the far region of the tail. As a consequence we get VaR estimates that we compare with the empirical ones. The analysis has to be taken with care because of the intricate dependency of financial data.

Keywords: Financial Time Series, Value at Risk, Extreme Value Theory, Peaks over Thresholds.

1 Introduction

Recently there has been great interest in the applications of the mathematical theory of extremal events to problems in insurance and finance. In particular it well suits the field of risk management, especially the part concerning market risk. Extreme value theory (EVT) provides with methods to estimate probability distributions of risky portfolios and to calculate various measures of risk as VaR and the so called expected shortfall, which was proposed by Artzner et al. (1998) as a "coherent" measure of risk. There is also an increasing demand of mathematical and statistical methods for credit risk modelling and EVT promises to play an important role on this matter too.

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The aim of this note is to illustrate the basic tools and methods of EVT through an analysis of Italian financial data. In particular we will apply the Peaks over Threshold (POT) method to model the tail of the distribution of both losses and gains. We think that the POT method is one of the main methods of the theory and it shows well strenghts and weakness of it. The data we consider are the daily logarithmic returns of the FIAT share. We will show how to get a satisfactory modelling in the tail of the distribution. Together with the fitting we obtain estimates for quantiles, i.e. for VaR, just as the inverse of our models distribution function at the given probability level. However it is important to remark that, due to the intricate dependency of financial data, the analysis done here has to be thought of as a first description, because our results are based on the assumption of indipendence. Recent research has been done to analyze more deeply the problem of dependency in finance with an EVT approach (see for instance the works of Mc Neil (1998b) and Frey and McNeil (1998)).

The main feature of EVT is to provide with different natural models for extremal events. In other words it gives methods to deal with heavy tail distributions, especially in the far region of the tail where the data are very large and therefore rare. As we will see with the FIAT share, Italian data do not show particularly fat tails (other assets could also be analyzed with same type of results).

The paper is structured as follows: in the first section we briefly give the main theoretical results of extreme value theory needed and in the second section we illustrate in some detail the POT method. The third section is devoted to the study of the data, beginning with some exploratory analysis (§4.1), followed by some models for the tails of the distribution above different suitable values. We will concentrate more on the left tail, i.e. on losses. In the last section we obtain some quantile estimates from our models and we compare them with the empirical ones.

The analysis has been performed using the software EVIS (Extreme Values in S-Plus) written by Dr. A. McNeil which can be downloaded from http://www.math.ethz.ch/ ~mcneil.

2 Extreme value theory

In this section we present a brief overview of the main results of extreme value theory with special focus on the part that will be applied to our data. For a general treatment of the subject see Embrechts, Klüpperberg, Mikosch (1997) to which we will often refer for more detailed results and discussions. In this book one can also find an extensive list of references. We will treat and use only the theory under the assumption of indipendence and identical distribution, but we would like to mention that the theory works also under more general hypotheses (e.g., Embrechts, Klüpperberg, Mikosch, 1997, §4.4 and Ch.8).

2.1 The generalised extreme value distributions

Let $X_1, X_2, \ldots$ be a sequence of indipendent, identically distributed (i.i.d.) random variables with distribution function F, i.e. $P(X_i \leq x) = F(x)$ for every i. Let $M_n = \max(X_1, \ldots, X_n)$ so that $P(M_n \leq x) = F^n(x)$. We say that F is in the *maximum domain of attraction* of H and we write

$$F \in \mathrm{MDA}(H)$$

if there exists a sequence of real numbers $a_n > 0$ and b_n such that the sequence of normalized maxima $(M_n - b_n)/a_n$ converges in distribution to some non-degenerate random variable , i.e.

$$P((M_n - b_n)/a_n \leq x) = F^n(a_n x + b_n) \stackrel{n\to\infty}{\longrightarrow} H(x),$$

for some non-degenerate distribution function $H(x)$ (this condition is equivalent to convergence in distribution because it can be shown that all possible limit distribution functions are continuous). Note that this is a similar setting to the central limit theorem, where one studies the asymptotic behaviour of sums instead of maxima.

Let us introduce now a family of distribution functions which goes under the name of generalised extreme value distribution (GEV).

$$H_\xi(x) = \begin{cases} \exp(-(1+\xi x)^{-1/\xi}) & \xi \neq 0, \\ \exp(-e^{-x}) & \xi = 0 \end{cases}$$

with $1 + \xi x > 0$. ξ is called the shape parameter. It is useful to introduce also a scale and a location parameter $\psi > 0$ and μ by considering the three parameter family

$$H_{\xi,\psi,\mu}(x) := H_\xi((x-\mu)/\psi).$$

One says that two random variables X and Y (or their distribution functions) *are of the same type* if there exist constants $\mu \in \mathbb{R}$ and $\psi > 0$ such that X and $\psi Y + \mu$ have the same distribution. Therefore $H_{\xi,\psi,\mu}$ is of the same type as H_ξ.

If $\xi > 0$ the GEV distribution is also called Fréchet, for $\xi = 0$ Gumbel and for $\xi < 0$ Weibull. We will see below that this class of functions plays a role analogous to the α-stable distributions in the central limit theorem. In this setting the basic theorem is due to Fisher-Tippet (1928):

Theorem *If $F \in MDA(H)$ then H is of the same type as H_ξ for some ξ.*

A sketch of the proof can be found in Embrechts, Klüpperberg, Mikosch (1997), on page 122. The Fisher-Tippet theorem classifies all the possible (non-degenerate) limit distributions of suitably normalized maxima of i.i.d. random variables. It is important to notice that essentially all common continuous distributions are in the maximum domain of attraction of some extreme value distribution, as we will briefly see below (see also Embrechts, Klüpperberg, Mikosch (1997), pp. 153-157, for detailed lists). An important issue is to characterize the MDA(H_ξ) for all ξ. Very important for applications in finance and insurance is the maximum domain of attraction of H_ξ for $\xi > 0$, i.e. the Fréchet case. In this case a theorem by Gnedenko (1943) gives a complete answer to the problem. He showed that for $\xi > 0$

$$F \in \mathrm{MDA}(H_\xi) \Longleftrightarrow 1 - F(x) = x^{-1/\xi} L(x)$$

for some slowly varying function $L(x)$ (Embrechts, Klüpperberg, Mikosch (1997), §3.3.1). A function L on $(0, \infty)$ is said to be *slowly varying* if

$$\lim_{x \to \infty} \frac{L(tx)}{L(x)} = 1 \text{ for every } t > 0.$$

Examples of slowly varying functions are the constant functions, logarithm or functions that behave asymptotically like constants or logarithms. The result then essentially says that if the tail of the distribution function F decays like a power function then F belongs to the maximum domain of attraction of the Fréchet and viceversa. So the class of distribution in the domain of attraction of the Fréchet includes the so called heavy tailed distributions and the parameter ξ measures the fatness of the tail. Not all the moments of such distributions are finite. These type of distributions are often encountered in modelling losses in insurance and finance. Examples are the Pareto, the Cauchy and the t-distributions.

Also important is the case of the Gumbel, $\xi = 0$. The characterization of this domain of attraction is more complicated (Embrechts, Klüpperberg, Mikosch (1997), §3.3.3). Essentially it contains distributions with tail decaying exponentially and with all finite moments. These function may be referred to as light tailed. Important examples are the normal, the lognormal and the exponential.

The Weibull contains in its domain distributions with bounded support like the uniform and the beta. Like in the Fréchet case this domain is well characterized, but we will not give the exact statement here, since we will treat only distributions with unbounded support. For an extensive discussion about this domain we refer again to Embrechts, Klüpperberg, Mikosch (1997), §3.3.2.

The Fisher-Tippet theorem can be exploited in practice to analyze sample maxima. It has important applications in hydrology, where one is interested typically in annual maxima (e.g. temperature, level of a river). It also naturally gives a method to construct stress scenarios. In a financial context there are works of Longin (1997) and McNeil (1998a), (1998b) (in the latter the analysis takes into account also the dependence structure and the so called extremal index).

2.2 The generalised Pareto distribution

For our application it is important the following class of functions, called generalised Pareto distributions (GPD).

$$G_\xi(x) = \begin{cases} 1-(1+\xi x)^{-1/\xi} & \xi \neq 0, \\ 1-e^{-x} & \xi = 0 \end{cases}$$

with

$$\begin{array}{rl} x \geq 0 & \text{if } \xi \geq 0, \\ 0 \leq x \leq -1/\xi & \text{if } \xi < 0. \end{array}$$

As in the case of the GEV one considers the three parameter family

$$G_{\xi,\beta,\nu}(x) := G_\xi\left(\frac{x-\nu}{\beta}\right).$$

with $\beta > 0$.

Under the assumption that our model F belongs to some maximum domain of attraction, the GPD will estimate the probability distribution of the exceedances, given that a certain amount was exceeded. This stands as a useful results by itself, but it will also be used to estimate the tail of the distribution. More precisely we denote by x_F the right end point of F, i.e. $x_F = \sup\{x \in \mathbb{R} \ : \ F(x) < 1\}$. For our application one can think $x_F = \infty$. Define, for a fixed $u < x_F$,

$$F_u(x) := P(X - u \leq x \mid X > u), \quad x \geq 0$$

the *excess distribution function.*

Let us state the basic result:

Theorem (Pickands (1975), Balkema-de Haan (1974)) *For every* $\xi \in \mathbb{R}$, $F \in \mathrm{MDA}(H_\xi)$ *if and only if*

$$\lim_{u \nearrow x_F} \sup_{0<x<x_F-u} \mid F_u(x) - G_{\xi,\beta(u)}(x) \mid = 0$$

for a positive measurable function $\beta(u)$.

The proof can again be found in Embrechts, Klüpperberg, Mikosch (1997) on page 165. The theorem basically says that if we choose a sufficiently high threshold u and a suitable β we will have that $F_u(x) \approx G_{\xi,\beta}(x)$, under the hypothesis that $F \in \mathrm{MDA}(H_\xi)$. Of course in the applications we don't know ξ and we need to estimate it.

3 The POT method

Typically we have historical data (e.g. losses) $X_1, \ldots, X_n$ that we suppose indipendent and identically distributed. One can look at the empirical distribution function, namely $F_n(x) = \frac{\#\{i\,:\,i=1,\ldots,n\,,\,X_i \leq x\}}{n}$, which is a good approximation of the central part of the distribution, but it is not anymore when there are only few data above a certain level. Moreover we are mostly interested in modelling exactly those few extremal data. Let us suppose now that our sample comes from a distribution function $F \in \mathrm{MDA}(H_\xi)$. As we noticed, this hypothesis is quite general. Let us fix a threshold u and let

$$N_u = \#\{i \,:\, i = 1, \ldots, n\,,\, X_i > u\}$$

the number of exceedances of u. For $x \geq u$ we can write

$$(1 - F(x)) = (1 - F(u))(1 - F_u(x-u))$$

This simple relation suggests an estimator of the tail of the distribution, for $x \geq u$: first estimate empirically $1 - F(u)$ as $\frac{N_u}{n}$. Notice that one would need u sufficiently small to have a reasonable number of data. For the second part one can invoke the Pickands-Balkema-de Haan theorem so that the resulting estimate for the tail is:

$$(1 - \hat{F}(x)) = \frac{N_u}{n}\left(1 - G_{\hat{\xi},\hat{\beta},u}(x)\right) = \frac{N_u}{n}\left(1 + \hat{\xi}\frac{(x-u)}{\hat{\beta}}\right)^{-1/\hat{\xi}}$$

for appropriate $\hat{\xi}, \hat{\beta}$, which of course depend on u and have to be estimated from the exceedances over u. Here the threshold u should be also sufficiently large, in order to apply the Balkema-Pickands-de Haan theorem. One of the crucial points of the theory is indeed the choice of a good threshold.

It is easy to see that $\hat{F}(x)$ is a GPD $G_{\xi,\beta',\nu}(x)$ with same shape parameter ξ and scale and location parameters given by

$$\begin{cases} \hat{\beta}' & = (\frac{N_u}{n})^{\hat{\xi}}\hat{\beta} \\ \hat{\nu} & = u - \frac{\hat{\beta}}{\hat{\xi}}\left(1 - (\frac{N_u}{n})^{\hat{\xi}}\right) \end{cases}$$

The method gives then a way to estimate the distribution F above a certain level u. Basically one fits a three parameter Pareto distribution to F, but the strength of the method comes also from its theoretical justification. One can say that this estimate uses the whole distribution to extrapolate beyond the data, in the region where there is not much information because the events are rare. At this point one gets also an estimate for the quantile $x_p = F^{\leftarrow}(p) := \inf\{x \in \mathbb{R} \,:\, F(x) \geq p\}$ (for strictly increasing F this is just the inverse function), if $p > F(u)$, simply as the inverse of the GPD calculated at the desired probability level p:

$$\hat{x}_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right)$$

To estimate the shape and scale parameters $\hat{\xi}, \hat{\beta}$ one can use classical methods like maximum likelihood or the probability weighted moments. The method adopted in the software we used is maximum likelihood. In any case one has:

$$\begin{aligned}\hat{\xi} &= \hat{\xi}_{N_u}\left(X_1 - u, \ldots, X_{N_u} - u\right), \\ \hat{\beta} &= \hat{\beta}_{N_u}\left(X_1 - u, \ldots, X_{N_u} - u\right),\end{aligned}$$

where $(X_1 - u, \ldots, X_{N_u} - u)$ are suppose to come from a GPD $G_{\xi,\beta}$. It is not difficult to write the log-likelihood function which can be solved numerically. For $\xi > -1/2$ it can be shown that maximum likelihood regularity conditions are satisfied and that the estimates are asimptotically normal, which gives also the possibility to calculate approximate standard errors and confidence intervals (see Embrechts, Klüpperberg, Mikosch (1997), pp. 356-358, for a more extensive discussion and for references).

4 The analysis of the FIAT returns

The data are the FIAT logarithmic returns (henceforth returns) from February 14, 1986 to October 4, 1999. We are mainly interested in the negative returns, i.e. in losses. However we will model both the negative and the positive ones, namely left and right tail of the entire distribution. As already mentioned in the introduction the negative returns are not very heavy tailed, but yet with power decay, while the positive ones appear even quite light tailed (exponential decay). To study the losses we take absolute values of the negative returns. The resulting number of observations for losses is 1724 and for gains is 1683. A plot of the time series of prices is given in Figure 1 (top), while a plot of returns is given in Figure 1 (bottom). In Figure 2 are shown the histograms (top) and the empirical distribution functions (bottom) of losses (left) and of gains (right).

4.1 Exploratory data analysis

The main exploratory tools used in extreme value theory are the quantile-quantile plot (QQ-plot) and the sample mean excess plot (ME-plot). The QQ-plot makes us compare in a graphical way our data with a model F. Let $X_{n,n} \leq X_{n-1,n} \leq \ldots \leq X_{1,n}$ be the ordered data. One plots the following

$$\left\{\left(X_{k,n}, F^{\leftarrow}\left(\frac{n-k+1}{n+1}\right)\right) \; k = 1, \ldots, n\right\}$$

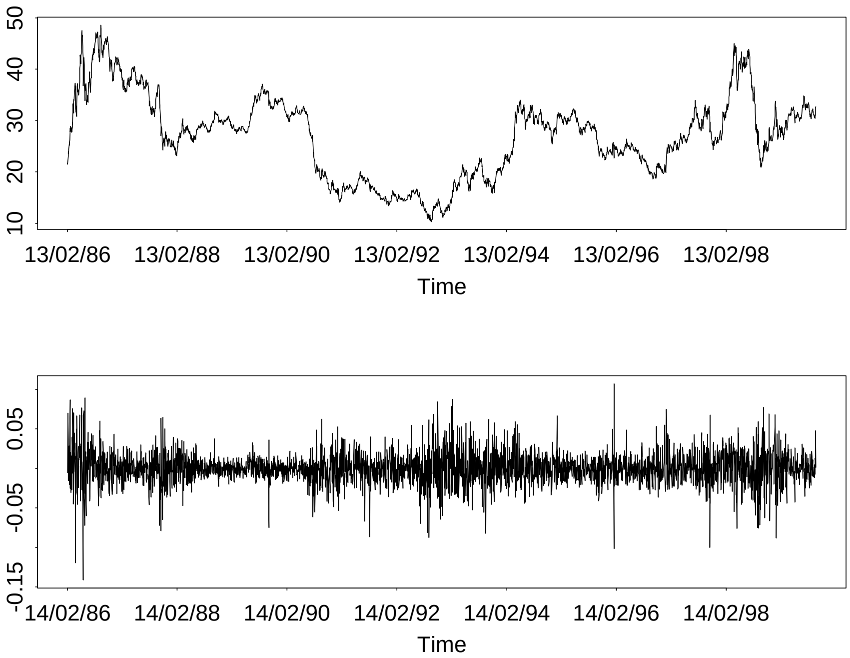

Figure 1: FIAT prices from 13/2/86 to 4/10/99 (top) and FIAT returns from 14/2/86 to 4/10/99 (bottom).

Basically one compares the quantiles of the empirical distribution function F_n with the quantiles of the reference model F. A good fit shows approximately a line in the plane. If the plot curves down to the right the data have a right tail which is heavier than the model F and if it curves up to the left the data have a heavier left tail. This is the case of the QQ-plot of all the FIAT returns against the normal (Figure 3 (top left)). It illustrates what expected: the normality assumption is not conceivable for returns of financial assets, which are much heavier, especially losses. The second (Figure 3 (top right)) shows the QQ-plot of the negative returns against the exponential. The data look only a bit heavier than the exponential. We see here already, which we will make more precise later, that our Italian data are not so heavy tailed as for instance the BMW returns studied in Embrechts, Klüpperberg, Mikosch (1997), on pages 299, 360, 361. The plot is indeed only slightly curving down. At first sight one could even think about fitting an exponential, but there are anyhow signs of heavier tail (see Figure 3 (bottom left)).

Let us define now the mean excess function of a random variable X with right end point x_F as $e(u) := E(X - u \mid X > u)$ for $u < x_F$. It is possible to compute $e(u)$ (see again Embrechts, Klüpperberg, Mikosch (1997), §6.2.2, for much more details) for different distributions: for the GPD $G_{\xi,\beta}$ ($\xi < 1$ so that $e(u)$ exists), then $e(u) = \frac{\beta+\xi u}{1-\xi}$ (for the exponential $G_{0,\beta}$ $e(u) = \beta$). One can

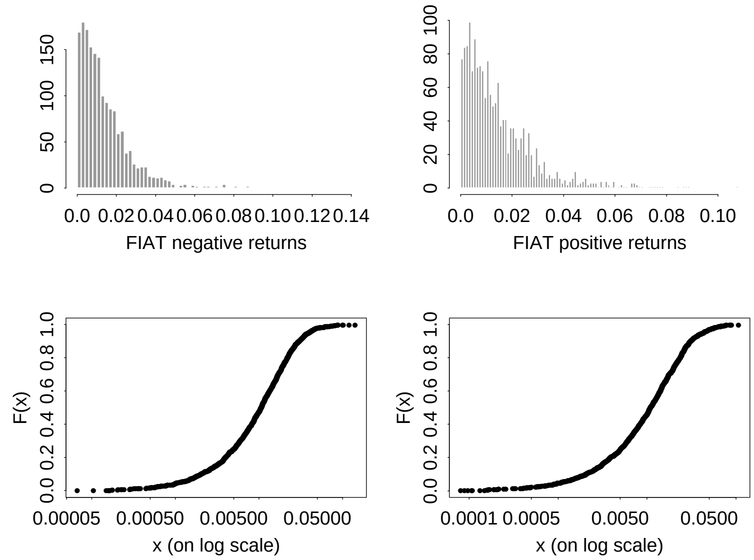

Figure 2: Histograms (top) and empirical distribution functions (bottom) of losses (left) and gains (right)

show that for the normal $e(u)$ goes to zero as u goes to infinity. So we see that the heavy tail behaviour presents a ME-plot with an upward trend, a medium tail shows a horizontal line and for a light tail goes down to small values. One can naturally define the empirical mean excess function

$$e_n(u) := \frac{1}{\#\{i\,:\,X_i > u\}} \sum_{i:X_i>u} (x_i - u)$$

and plot the graph $\{(X_{k,n}, e_n(X_{k,n}))\, k = 1, \ldots, n\}$. It is suggested to try some plot of simulations to realize how the empirical plot could look different from the analytic one (see Embrechts, Klüpperberg, Mikosch (1997), p. 297, or Figure 4 (bottom pictures)). The use of the ME-plot in the POT method, i.e. in the fitting of a GPD to our data, is basically the following: one tries to look at a threshold u from which the plot presents approximately a straight line behaviour, as it is the case for a GPD. This task is not completely obvious and one has to give different tries. Change in concavity of the plot could also be appropriate. Here are the ME-plot of FIAT negative (Figure 4 (top left)) and positive (Figure 4 (top right)) returns. It is clear how the negative ones present a plot going upward (except for smaller values of u. This however seems to be a typical behaviour for financial data), while the positive ones present a more constant behaviour. In Figure 4 (bottom left) there is the ME-plot of 1000 data simulated from a GPD with shape parameter $\xi = 0.15$ and in Figure 4 (bottom right) the ME-plot of 1000 data simulated from an exponential, to

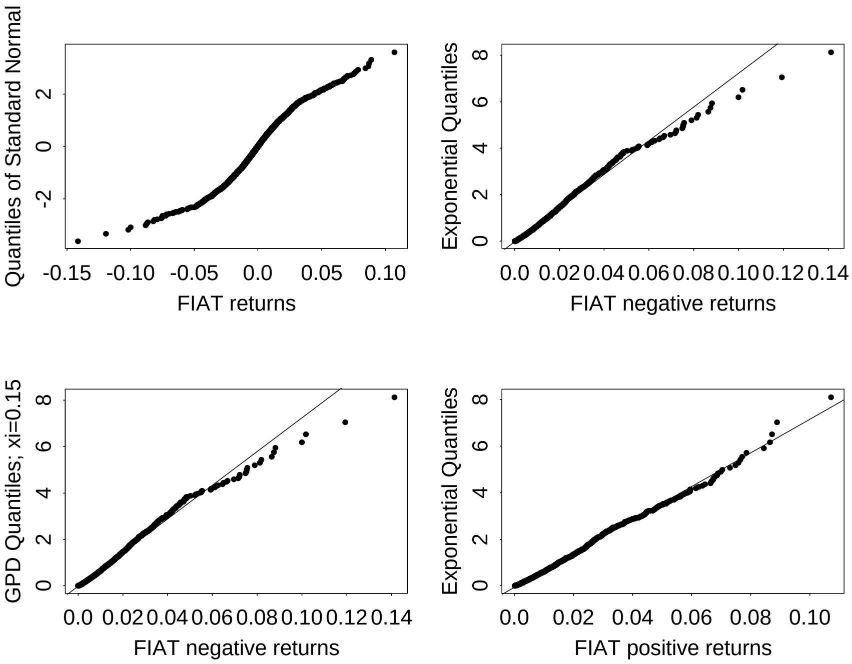

Figure 3: Top left: QQ-plot of all FIAT returns against the normal. Top right: QQ-plot (of the absolute value) of negative returns against the exponential. Bottom left: QQ-plot (of the absolute value) of negative returns against a GPD with $\xi = 0.15$. Bottom right: QQ-plot of FIAT positive returns against the exponential.

compare with those of our real data.

4.2 Modelling the tail

We want to remark again that our goal is to model the tails of the distribution for large values, above some suitable threshold u. For the body of the distribution the empirical function would work fine. Also we are interested in computing high quantiles and for that we need to look at the end of the tail. Our GPD model is not designed to work below u and indeed it will not (see Figures 7 and 8). Let us concentrate on the negative returns. The ME-plot suggests to try values for the threshold in the range 0.017-0.034 like 0.017, 0.02, 0.026, 0.032. We will show also some fitting of the GPD to the excess distribution $F_u(x - u)$.

The following figure (Figure 5) illustrates the estimates of the shape parameter ξ as a function of the increasing threshold in the upper axis and of the respective decreasing number of exceedances in the lower axis. This is done, as said before, using maximum likelihood estimations on the exceedances, which

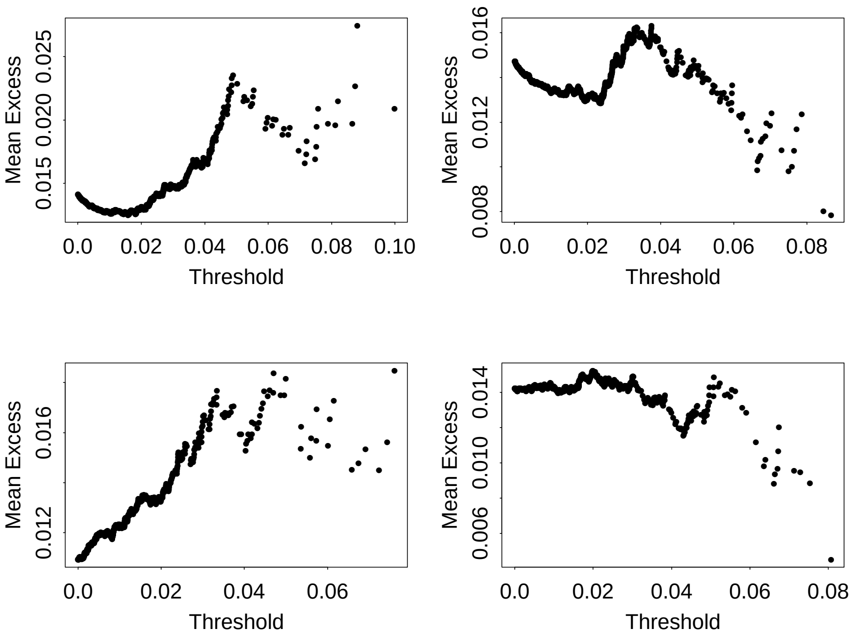

Figure 4: ME-plot of FIAT negative returns (top left), positive returns (top right), 1000 simulated data from a GPD with $\xi = 0.15$, $\beta = 0.009$, $\nu = 0$ (bottom left), 1000 simulated data from an exponential with $\beta = 0.014$, $\nu = 0$ (bottom right).

also gives the specified confidence interval (0.95 C.I.) for the estimate. We see that the estimate is quite stable as the value of the threshold changes.

Figure 6 shows the results of the POT method corresponding to different values of the threshold u. For each u we show first the fitting of the $G_{\hat{\xi},\hat{\beta},u}(x)$ to the empirical excess distribution, then the fitting of the $G_{\hat{\xi},\hat{\beta}',\hat{\nu}}(x)$ to the tail. Figure 7 shows only the fitting to the tail.

We show different fittings, with different thresholds and different values of the main shape parameter. A lower ξ gives a more optimistic model, lighter tailed, while the higher value 0.174 of the model at the bottom of Figure 6 is more conservative. Then one can look at the corresponding quantile estimates in the next section. One should take into account the entire picture, not only a particular model. However we will see that the quantile estimates are quite stable.

Now let us very briefly discuss the right tail of the FIAT returns, i.e. the distribution of gains. One can go through the same type of analysis as for the negative ones. As shown in Figure 4 (top right) the ME-plot suggests a lighter tail behaviour, somehow exponential. The QQ-plot against the exponential (Figure 3 (bottom right)) confirms this idea. One could also try to plot the shape parameter (the analogue of Figure 5) and see that indeed the various

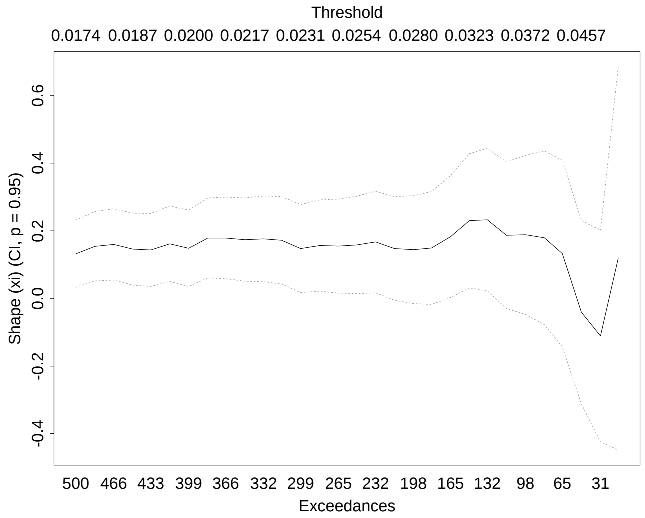

Figure 5: Shape parameter ξ for the negative returns as a function of the increasing threshold (upper axis) and of the decreasing number of exceedances (lower axis). 30 models are fitted.

estimates give values for ξ around zero. Figure 8 shows a model for the tail above $u = 0.027$. First we fitted a $G_{0,\beta}$ (i.e. an exponential) to the exceedances and then we multiplied by the factor $\frac{N_u}{n}$ as suggested in the POT method. Again the model does not give a good fit below u.

5 Quantile estimates

Here we give results only for losses. Figure 9 shows graphically the 0.95 and the 0.99 quantile estimates. The estimates give values respectively of 0.0392, 0.0648 with 95% confidence intervals for the estimates [0.0374, 0.0415], [0.0593, 0.0727]. We calculated also the 0.995 and the 0.999 quantile estimates for this model which are respectively 0.0778 and 0.1138 with 95% confidence intervals [0.0696, 0.091] and [0.0945,0.1496]. We choose our favorite model $G_{\hat{\xi},\hat{\beta}',\hat{\nu}}$ for the tail above a certain threshold u (in particular we picked the one corresponding to $u = 0.02$, with estimate for $\xi = 0.149$, somehow intermediate). Analytically one computes the quantile using the formula in §3. Graphically this corresponds to drawing the horizontal line (which is not done in the figure) at the desired probability levels (0.05 and 0.01) and then picking the corresponding value on the x-axis. The confidence interval was obtained using a profile likelihood method and in the graph corresponds to the downward curve.

In Figure 10 are shown 0.95 (top left), 0.99 (top right), 0.995 (bottom left) and 0.999 (bottom right) quantile estimates as functions of increasing

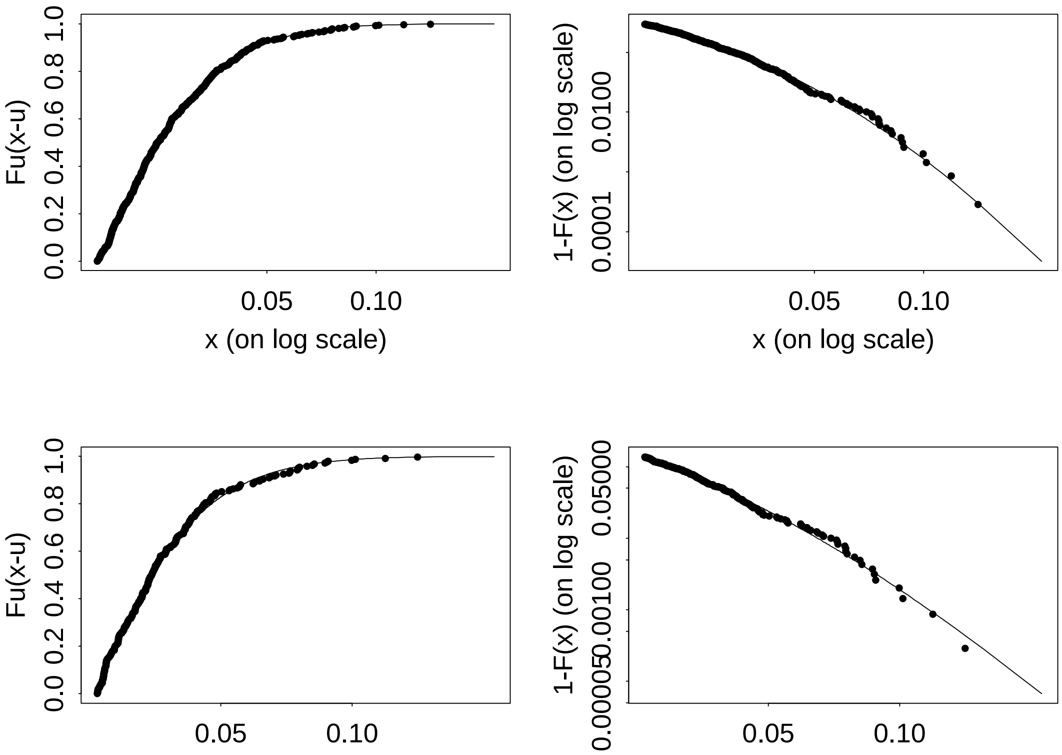

Figure 6: Negative returns. Top left: GPD fit to 515 exceedances of the threshold $u = 0.017$. The corresponding parameter estimates are $\xi = 0.128$, $\beta = 0.0112$. Top right: GPD fit to the tail above $u = 0.017$. The parameter estimates are $\xi = 0.128$, $\beta' = 0.00959$, $\nu = 0.00446$. Bottom left: GPD fit to 240 exceedances of the threshold $u = 0.026$. The corresponding parameter estimates are $\xi = 0.174$, $\beta = 0.0116$. Bottom right: GPD fit to the tail above $u = 0.026$. The parameter estimates are $\xi = 0.174$, $\beta' = 0.00823$, $\nu = 0.00663$.

thresholds (upper axis) and decreasing number of exceedances (lower axis), with 95% confidence intervals.

The pictures show that the estimates are fairly stable with the thresholds and therefore with the models, giving values respectively around 0.039, 0.065, 0.078 and 0.114.

Let us compare our estimates with the empirical estimator for quantiles which is just $X_{[n(1-p)+1],n}$. This gives 0.0393, 0.0696, 0.0811 and 0.1193 for p equal respectively to 0.95, 0.99, 0.995 and 0.999. Notice that for the last one we simply took the second highest observation, which means that there are only two observations above 0.1193.

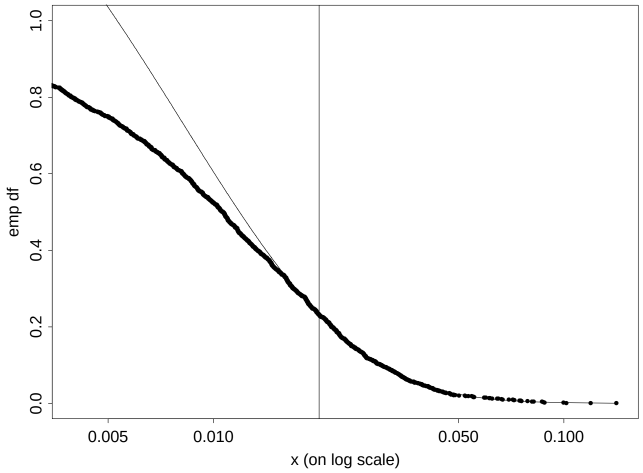

Figure 7: Negative returns. GPD fit to tail above $u = 0.02$. The parameter estimates are $\xi = 0.149$, $\beta' = 0.00898$, $\nu = 0.00531$. One sees that the model does not give a good fit below 0.02.

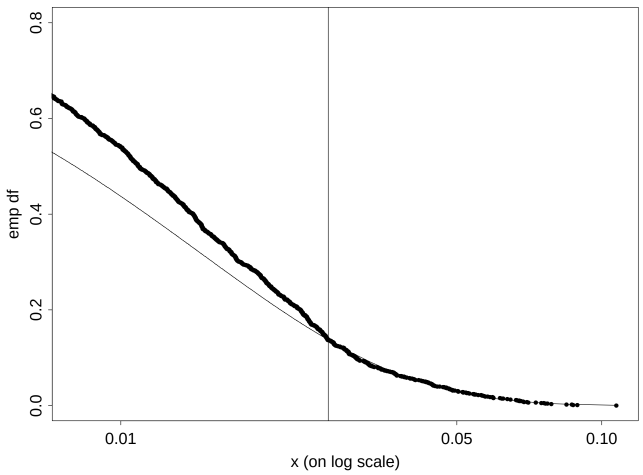

Figure 8: Positive returns. Model above $u = 0.027$. We took $\xi = 0$, while the estimate for β is 0.014.

6 Conclusions

We presented some methods of EVT to analyze financial data also with the scope of illustrating how powerful these methods are. Other techniques of the theory could be used, like the block maxima method giving the possibility of developing stress scenarios (e.g., Longin, 1997). One could also take into

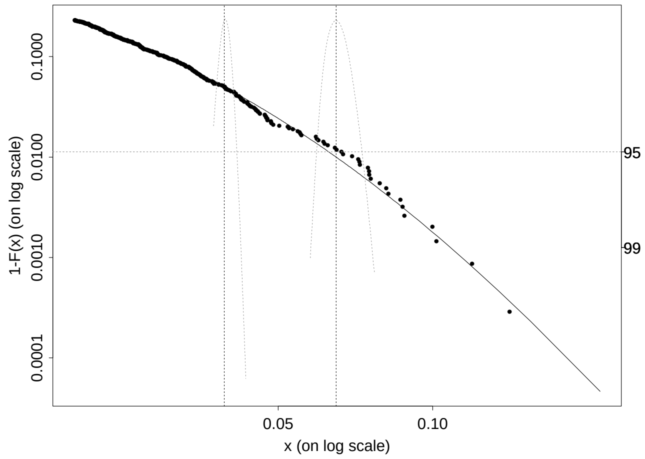

Figure 9: Negative returns. 0.95 and 0.99 quantile estimates corresponding to the GPD fitted to the tail above the threshold $u = 0.02$. The estimates are respectively 0.0392 and 0.0648 with confidence intervals [0.0374, 0.0415] and [0.0593, 0.0727].

account dependency and clustering within the framework of extremal events by introducing the extremal index (e.g., McNeil, 1998b). The software EVIS provides with functions performing these types of study. We would like to remark that we chose to show the FIAT share but also other Italian financial assets (MIB30, Olivetti, Generali, etc.) can be studied with satisfactory results with these tools. They appear to be quite light tailed in the same fashion as the FIAT returns.

We felt that this analysis is a good starting point and we hope that we demonstrated once again that EVT could play an important role in the field of risk management. However, as it is well understood, the intricate dependency of financial assets requires deeper investigations and new ideas. In this direction one can look for instance at the work of Frey and Mc Neil (1998) where GARCH modelling of financial series and EVT are combined. Also multivariate analysis has to be faced because one really deals with complex stochastic systems of high dimension.

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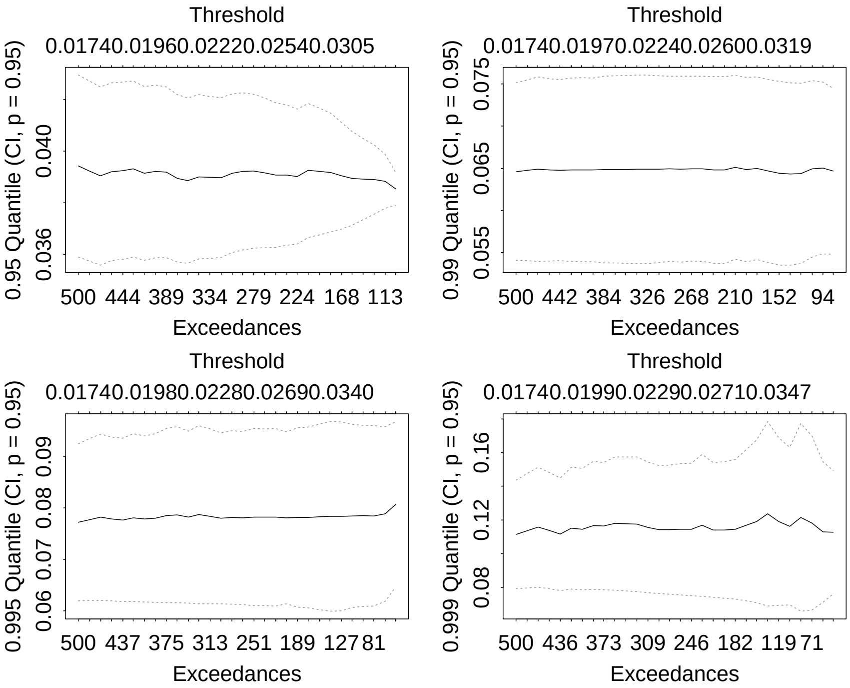

Figure 10: Negative returns. 0.95 (top left), 0.99 (top right), 0.995 (bottom left) and 0.999 (bottom right) quantile estimates as functions of increasing thresholds (upper axis) and decreasing number of exceedances (lower axis) with 95% confidence intervals.

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Riassunto

Lo scopo del lavoro è quello di illustrare dei metodi di teoria dei valori estremi attraverso un'analisi di dati finanziari Italiani. Ci concentriamo principalmente sul metodo cosiddetto POT (Peaks over Thresholds) e lo applichiamo ai rendimenti logaritmici giornalieri delle azioni FIAT dal 1986 al 1999. Si riesce ad ottenere una modellazione soddisfacente della distribuzione, sotto l'assunzione di indipendenza ed identica distribuzione, specialmente nella parte più lontana della coda. Conseguentemente si ottengono delle stime di VaR che vengono confrontate con le stime empiriche. L'analisi va presa con la dovuta cautela a causa della complessa dipendenza dei dati finanziari.