Nonmonotonic Extensions of Low Complexity DLs: Complexity Results and Proof Methods

Laura Giordano[1], Valentina Gliozzi[2], Nicola Olivetti[3], and Gian Luca Pozzato[2]

[1] Dipartimento di Informatica - Università del Piemonte Orientale “A. Avogadro”

[2] Dipartimento di Informatica - Università edgli Studi di Torino

[3] LSIS-UMR CNRS 6168 Universitè “Paul Cézanne” - Aix-Marseille 3

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- A non-monotonic extension of the low complexity (lightweight) Description Logics $\mathcal{EL}^{\perp}$ and DL-lite$_c$ for reasoning about prototypical properties and inheritance with exceptions

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- Basic idea: to extend DLs with a typicality operator $\mathbf{T}$
- $\mathbf{T}(C)$ singles out the “most normal” instances of the concept C

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- to handle defeasible inheritance needs the integration of some kind of nonmonotonic reasoning mechanism [BH95, BLW06, DLN$^+$98, DNR02, ELST, Str93]
- However, all these methods present some difficulties

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  - DL + a typicality operator $\mathbf{T}$
  - meaning of $\mathbf{T}$: (for any concept C) $\mathbf{T}(C)$ singles out the "typical" instances of C
- semantics of $\mathbf{T}$ defined by a set of postulates that are a restatement of Kraus-Lehmann-Magidor axioms of preferential logic **P** (Representation Theorem [GGOP09])

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- A KB comprises, in addition to the standard TBox and ABox, a set of assertions of the type $\mathbf{T}(C) \sqsubseteq D$ where D is a concept not mentioning $\mathbf{T}$
- "normally students do not pay taxes" $\Rrightarrow$ $\mathbf{T}(Student) \sqsubseteq \neg TaxPayer$
- Example: normally a student does not pay taxes, normally a working student pays taxes, but normally a working student having children does not pay taxes (because he is discharged by the government)

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$$\mathbf{T}(Student) \sqsubseteq \neg TaxPayer$$
$$\mathbf{T}(Student \sqcap Worker) \sqsubseteq TaxPayer$$
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- Which inferences?

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Logic of typicality

- We have defined a nonmonotonic inference based on a minimal model semantics
- For $DL + \mathbf{T} = \mathcal{ALC} + \mathbf{T}$ nonmonotonic inference has a high complexity, namely $\textsc{co-NExp}^{\mathrm{NP}}$, comparable however with that one of other NMR DL (circumscription)
- We are interested in applying our approach to low-complexity DLs $\mathcal{EL}^{\perp}$ and DL-Lite$_{core}$.

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  - reasoning in $\mathcal{EL}$ is polynomial-time decidable

Language of $\mathcal{EL}^{\perp}\mathbf{T}_{\mathbf{min}}$

- Alphabet of
  - concept names $\mathcal{C}$
  - role names $\mathcal{R}$
  - individuals $\mathcal{O}$
- Given $A \in \mathcal{C}$ and $r \in \mathcal{R}$, we define:

$$\begin{aligned} C &:= A \mid \top \mid \bot \mid C \sqcap C \\ C_R &:= C \mid C_R \sqcap C_R \mid \exists r.C \\ C_L &:= C_R \mid \mathbf{T}(C) \end{aligned}$$

- TBox contains a finite set of concept inclusions $C_L \sqsubseteq C_R$

Example

The reformulation of the previous example in $\mathcal{EL}^{+^\perp}\mathbf{T}$ gives the following KB:

$$TaxPayer \sqcap NotTaxPayer \sqsubseteq \bot$$

$$Parent \sqsubseteq \exists HasChild.\top$$

$$\exists HasChild.\top \sqsubseteq Parent$$

$$\mathbf{T}(Student) \sqsubseteq NotTaxPayer$$

$$\mathbf{T}(Student \ \sqcap \ Worker) \sqsubseteq TaxPayer$$

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Language of DL-Lite$_c$T$_{min}$

- Alphabet of
  - concept names $\mathcal{C}$
  - role names $\mathcal{R}$
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- Given $A \in \mathcal{C}$ and $r \in \mathcal{R}$, we define:

$$
\begin{aligned}
C_L \quad &:= A \mid \exists R.\top \mid \mathbf{T}(A) \\
R \quad &:= r \mid r^- \\
C_R \quad &:= A \mid \neg A \mid \exists R.\top \mid \neg \exists R.\top
\end{aligned}
$$

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Monotonic Semantics

A model $\mathcal{M}$ is a structure $\langle \Delta, <, I \rangle$, where Δ is the domain and

- for each extended concept C, $C^I \subseteq \Delta$, and for each role R $R^I \subseteq \Delta \times \Delta$
- $<$ is an irreflexive and transitive relation over Δ satisfying the Smoothness Condition (well-foundness)
- $<$ is multilinear (or weakly connected): if $u < z$ and $v < z$, then either $u = v$ or $u < v$ or $v < u$

Semantics of the $\mathbf{T}$ operator: $(\mathbf{T}(C))^I = Min_<(C^I)$. For the other operators C^I is defined in the usual way (in particular, $(r^-)^I = \{(a, b) \mid (b, a) \in r^I\}$)

A model satisfying a Knowledge Base (TBox,ABox) is defined as usual

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  - $(\Box C)^I = \{x \in \Delta \mid$ for every $y \in \Delta$, if $y < x$ then $y \in C^I\}$
- Thus $\mathbf{T}(C)^I = (C \sqcap \Box\neg C)^I$

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- $\mathcal{EL}^{+^{\perp}}\mathbf{T}$ allows one to reason about typicality
- e.g. we can consistently express that student, working student and working student with children have a different status as taxpayers
- but we cannot derive anything about the prototypical properties of a given individual, unless the KB contains explicit tipicality assumptions concerning this individual

Weakness of monotonic semantics

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  - If $(Student \sqcap Worker \sqcap Parent)(john) \in$ ABox, we cannot derive $NotTaxPayer(john)$

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- Informally, we prefer a model $\mathcal{M}$ to a model $\mathcal{N}$ if $\mathcal{M}$ contains more typical instances of concepts than $\mathcal{N}$
- Given a KB, we consider a finite set $\mathcal{L}_T$ of concepts occurring in the KB, the typicality of whose instances we want to maximize
- $\mathcal{L}_T$ contains at least all concepts C such that $\mathbf{T}(C)$ occurs in the KB or in the <u>query</u>

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- Given two models $\mathcal{M} = \langle \Delta_{\mathcal{M}}, <_{\mathcal{M}}, I_{\mathcal{M}} \rangle$ and $\mathcal{N} = \langle \Delta_{\mathcal{N}}, <_{\mathcal{N}}, I_{\mathcal{N}} \rangle$ of KB, we say that $\mathcal{M}$ is preferred to $\mathcal{N}$ w.r.t. $\mathcal{L}_T$ ($\mathcal{M} <_{\mathcal{L}_T} \mathcal{N}$), if:

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  - $\mathcal{M}^{\Box^-}_{\mathcal{L}_T} \subset \mathcal{N}^{\Box^-}_{\mathcal{L}_T}$
  - $a^I = a^{I'}$ for all $a \in \mathcal{O}$
- A model $\mathcal{M}$ is a minimal model for KB (with respect to $\mathcal{L}_T$) if it is a model of KB and there is no a model $\mathcal{M}'$ of KB such that $\mathcal{M}' <_{\mathcal{L}_T} \mathcal{M}$

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- Query F : either a formula $C(a)$ or a subsumption $C \sqsubseteq D$
- Minimal Entailment in $\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}$
- A query F is minimally entailed from KB w.r.t. $\mathcal{L}_T$:
  KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$
  if F holds in all models of KB minimal w.r.t. $\mathcal{L}_T$

Example

Let $\mathcal{L}_{\mathbf{T}} = \{Student, Student \sqcap Worker, Student \sqcap Worker \sqcap Parent\}$

- $KB \cup \{Student(john)\} \models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} NotTaxPayer(john)$
- $KB \cup \{Student(john), Worker(john)\} \models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} TaxPayer(john)$
- $KB \cup \{Student(john), Worker(john), Parent(john)\} \models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} NotTaxPayer(john)$

Complexity results for $\mathcal{EL}^{\perp}\mathbf{T}_{\mathbf{min}}$

- Entailment for $\mathcal{EL}^{+^{\perp}}\mathbf{T}$ is CoNP , but
  - **Theorem 3.1 in [GGOP].** Entailment in $\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}$ is ExpTime-hard .
- We need further restrctions
- One possibility: Left Local $\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}$ (considered for circumscriptive extension [BLW06])

Language of Left Local $\mathcal{EL}^{\perp}\mathbf{T}_{\mathbf{min}}$

- Alphabet of
  - concept names $\mathcal{C}$
  - role names $\mathcal{R}$
  - individuals $\mathcal{O}$
- Given $A \in \mathcal{C}$ and $r \in \mathcal{R}$, we define:

$$
\begin{aligned}
C &:= A \mid \top \mid \bot \mid C \sqcap C \\
C_R &:= C \mid C_R \sqcap C_R \mid \exists r.C \\
C_L^{LL} &:= C \mid C_L^{LL} \sqcap C_L^{LL} \mid \exists r.\top \mid \mathbf{T}(C)
\end{aligned}
$$

- TBox contains a finite set of concept inclusions $C_L^{LL} \sqsubseteq C_R$

Complexity results for Left Local $\mathcal{EL}^{\perp}\mathbf{T}_{\mathbf{min}}$

- **Small model theorem (Theorem 3.11 in [GGOP]).** KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$ if and only if F holds in all models of KB whose size is polynomial in the size of KB.
- **Theorem 3.12 in [GGOP].** If KB is Left Local, the problem of deciding whether KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$ is in Π^p_2.

A small model theorem and a similar complexity result can be proved for DL-Lite$_c\mathbf{T}_{min}$ [GGOP]

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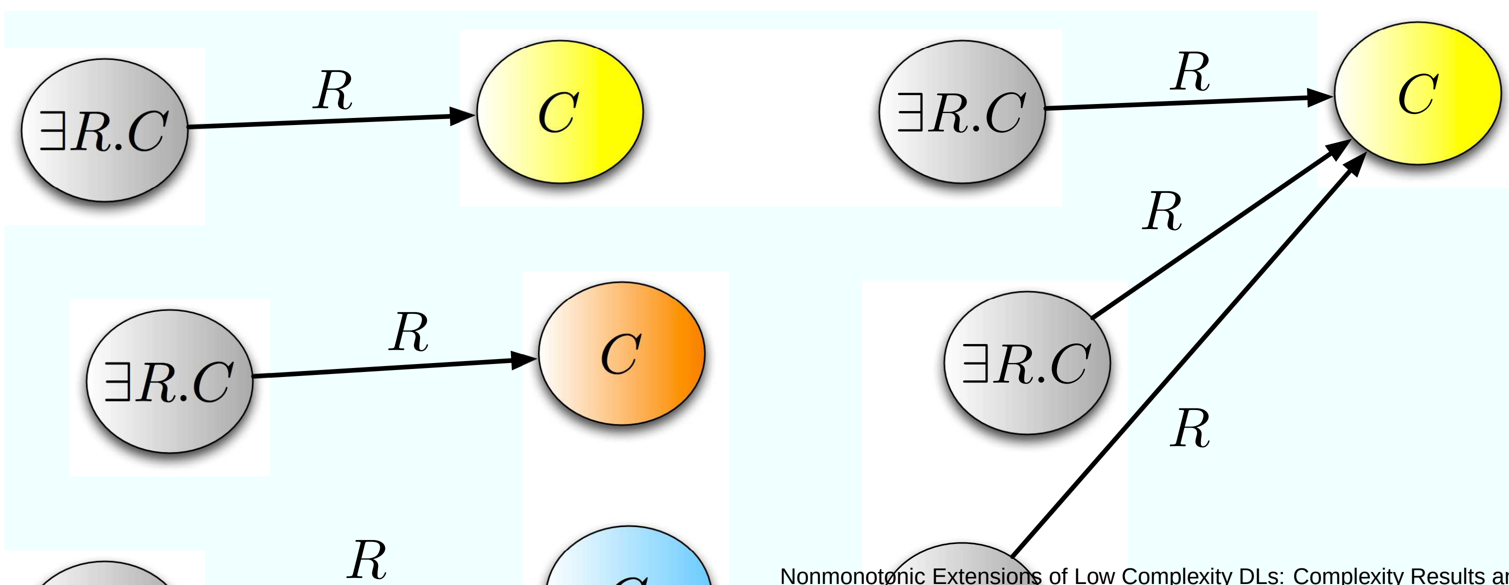
- **Small model theorem (Theorem 3.11 in [GGOP]).** KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$ if and only if F holds in all models of KB whose size is polynomial in the size of KB.
- **Theorem 3.12 in [GGOP].** If KB is Left Local, the problem of deciding whether KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$ is in Π_2^p.

A small model theorem and a similar complexity result can be proved for DL-Lite$_c\mathbf{T}_{min}$ [GGOP]



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    2. Phase 2: $\mathcal{TAB}_{PH2}^{\mathcal{EL}^{\perp}\mathbf{T}}$ checks whether the candidate models found in Phase 1 are minimal models of KB

The Tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$

- Given a knowledge base (TBox,ABox), tableaux nodes of $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ are called constraint systems and have the form $\langle S \mid U \mid W \rangle$, where :

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The Tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\bot}\mathbf{T}}$

- Given a knowledge base (TBox,ABox), tableaux nodes of $\mathcal{TAB}_{min}^{\mathcal{EL}^{\bot}\mathbf{T}}$ are called constraint systems and have the form $\langle S \mid U \mid W \rangle$, where :
  - $S = \{a : C \mid C(a) \in ABox\} \cup \{a \xrightarrow{R} b \mid aRb \in ABox\}$
  - $U = \{C \sqsubseteq D^{\emptyset} \mid C \sqsubseteq D \in TBox\}$

The Tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$

- Given a knowledge base (TBox,ABox), tableaux nodes of $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ are called constraint systems and have the form $\langle S \mid U \mid W \rangle$, where :
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  - W is a set of labels x_C used by existential rules

Special Existential Rules

The rule $(\exists^+)$ is split in the following two rules:

$$\frac{\langle S, u : \exists R.C \mid U \mid W \rangle}{\langle S, u \xrightarrow{R} x_C, x_C : C \mid U \mid W \cup \{x_C\} \rangle \quad \langle S, u \xrightarrow{R} y_1, y_1 : C \mid U \mid W \rangle \ \cdots \langle S, u \xrightarrow{R} y_m, y_m : C \mid U \mid W \rangle} (\exists^+)_1$$

if $x_C \notin W$ and $y_1, \ldots, y_m$ are all the labels occurring in S

$$\frac{\langle S, u : \exists R.C \mid U \mid W \rangle}{\langle S, u \xrightarrow{R} x_C \mid U \mid W \rangle \quad \langle S, u \xrightarrow{R} y_1, y_1 : C \mid U \mid W \rangle \cdots \langle S, u \xrightarrow{R} y_m, y_m : C \mid U \mid W \rangle} (\exists^+)_2$$

if $x_C \in W$ and $y_1, \ldots, y_m$ are all the labels occurring in S

Special Rule for $(\Box^-)$

$\overline{S} = S, u : \neg\Box\neg C_1, \ldots, u : \neg\Box\neg C_n$.
$S^M_{u\rightarrow y} = \{y : \neg D, y : \Box\neg D \mid u : \Box\neg D \in S\}$ and, for $k = 1, 2, \ldots, n$,
$\overline{S}^{\Box^{-k}}_{u\rightarrow y} = \{y : \neg\Box\neg C_j \sqcup C_j \mid u : \neg\Box\neg C_j \in \overline{S} \wedge j \neq k\}$.

$$\frac{\langle S, u : \neg\Box\neg C_1, \neg\Box\neg C_2, \ldots, u : \neg\Box\neg C_n \mid U \mid W\rangle}{\langle S, x : C_k, x : \Box\neg C_k, S^M_{u\rightarrow x}, \overline{S}^{\Box^{-k}}_{u\rightarrow x} \mid U \mid W\rangle \quad \langle S, y_1 : C_k, y_1 : \Box\neg C_k, S^M_{u\rightarrow y_1}, \overline{S}^{\Box^{-k}}_{u\rightarrow y_1} \mid U \mid W\rangle \cdots \langle S, y_m : C_k, y_m : \Box\neg C_k, S^M_{u\rightarrow y_m}, \overline{S}^{\Box^{-k}}_{u\rightarrow y_m} \mid U \mid W\rangle} \ (\Box^-)$$

for all $k = 1, 2, \ldots, n$, where $y_1, \ldots, y_m$ are all the labels occurring in S and x is new.
Rule $(\Box^-)$ contains:

- n branches, one for each $u : \neg\Box\neg C_k$ in $\overline{S}$;
- other $n \times m$ branches, where m is the number of labels occurring in S, one for each label y_i and for each $u : \neg\Box\neg C_k$ in $\overline{S}$

Phase 1: $\mathcal{TAB}_{PH1}^{\mathcal{EL}^{\perp}\mathbf{T}}$

$$\langle S, x : C, x : \neg C \mid U \mid W \rangle \ (\text{Clash}) \qquad \langle S, x : \neg\top \mid U \mid W \rangle \ (\text{Clash})_{\neg\top} \qquad \langle S, x : \bot \mid U \mid W \rangle \ (\text{Clash})_{\bot}$$

$$\frac{\langle S, x : C \sqcap D \mid U \mid W \rangle}{\langle S, x : C, x : D \mid U \mid W \rangle}(\sqcap^+) \qquad \frac{\langle S, x : \neg(C \sqcap D) \mid U \mid W \rangle}{\langle S, x : \neg C \mid U \mid W \rangle \quad \langle S, x : \neg D \mid U \mid W \rangle}(\sqcap^-) \qquad \frac{\langle S, x : C \sqcup D \mid U \mid W \rangle}{\langle S, x : C \mid U \mid W \rangle \quad \langle S, x : D \mid U \mid W \rangle}(\sqcup^+)$$

$$\frac{\langle S, x : \mathbf{T}(C) \mid U \mid W \rangle}{\langle S, x : C, x : \Box\neg C \mid U \mid W \rangle}(\mathbf{T}^+) \qquad \frac{\langle S, x : \neg\mathbf{T}(C) \mid U \mid W \rangle}{\langle S, x : \neg C \mid U \mid W \rangle \quad \langle S, x : \neg\Box\neg C \mid U \mid W \rangle}(\mathbf{T}^-) \qquad \frac{\langle S \mid U, C \sqsubseteq D^L \mid W \rangle}{\langle S, x : \neg C \sqcup D \mid U, C \sqsubseteq D^{L,x} \mid W \rangle}(\text{Unfold})$$

if x occurs in S and $x \notin L$

$$\frac{\langle S, u : \exists R.C \mid U \mid W \rangle}{\langle S, u \xrightarrow{R} x_C, x_C : C \mid U \mid W \cup \{x_C\} \rangle \quad \langle S, u \xrightarrow{R} y_1, y_1 : C \mid U \mid W \rangle \cdots \langle S, u \xrightarrow{R} y_m, y_m : C \mid U \mid W \rangle}(\exists^+)_1$$

if $x_C \notin W$ and $y_1, \ldots, y_m$ are all the labels occurring in S

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if $x_C \in W$ and $y_1, \ldots, y_m$ are all the labels occurring in S

$$\frac{\langle S, x : \neg\exists R.C, x \xrightarrow{R} y \mid U \mid W \rangle}{\langle S, x : \neg\exists R.C, x \xrightarrow{R} y, y : \neg C \mid U \mid W \rangle}(\exists^-)$$

if $y : \neg C \notin S$

$$\frac{\langle S \mid U \mid W \rangle}{\langle S, x : \neg\Box\neg C \mid U \mid W \rangle \quad \langle S, x : \Box\neg C \mid U \mid W \rangle}(cut)$$

if $x : \neg\Box\neg C \notin S$ and $x : \Box\neg C \notin S$

x occurs in S $\qquad C \in \mathcal{L}_{\mathbf{T}}$

$$\frac{\langle S, u : \neg\Box\neg C_1, \neg\Box\neg C_2, \ldots, u : \neg\Box\neg C_n \mid U \mid W \rangle}{\langle S, x : C_k, x : \Box\neg C_k, S^M_{u\to x}, \overline{S}^{\Box^{-k}}_{u\to x} \mid U \mid W \rangle \quad \langle S, y_1 : C_k, y_1 : \Box\neg C_k, S^M_{u\to y_1}, \overline{S}^{\Box^{-k}}_{u\to y_1} \mid U \mid W \rangle \cdots \langle S, y_m : C_k, y_m : \Box\neg C_k, S^M_{u\to y_m}, \overline{S}^{\Box^{-k}}_{u\to y_m} \mid U \mid W \rangle}(\Box^-)$$

x new

if $y_1, \ldots, y_m$ are all the labels occurring in $S, y_1 \neq u, \ldots, y_m \neq u$

$k = 1, 2, \ldots, n$

Phase 2: $\mathcal{TAB}_{PH2}^{\mathcal{EL}^{\perp}\mathbf{T}}$

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  - $\mathcal{D}($**B**$)$ as the set of labels occurring on **B**
  - **B**$^{\Box^-} = \{x : \neg\Box\neg C \mid x : \neg\Box\neg C$ occurs in **B**$\}$

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- K contains formulas of the form $x : \neg\Box\neg C$, with $C \in \mathcal{L}_T$
- Basic idea: given an open **B** built by $\mathcal{TAB}_{PH1}^{\mathcal{EL}^{\perp}\mathbf{T}}$, K is initialized with $\mathbf{B}^{\Box^-}$ in order to build smaller models

Phase 2: $\mathcal{TAB}_{PH2}^{\mathcal{EL}^{\perp}\mathbf{T}}$

$\langle S, x : C, x : \neg C \mid U \mid K \rangle$ (Clash)

$\langle S, x : \neg\top \mid U \mid K \rangle$ $(\mathrm{Clash})_{\neg\top}$

$\langle S, x : \bot \mid U \mid K \rangle$ $(\mathrm{Clash})_{\bot}$

$\langle S \mid U \mid \emptyset \rangle$ $(\mathrm{Clash})_{\emptyset}$

$\langle S, x : \neg\Box\neg C \mid U \mid K \rangle$ $(\mathrm{Clash})_{\Box^-}$ if $x : \neg\Box\neg C \notin K$

$$\frac{\langle S \mid U, C \sqsubseteq D^L \mid K \rangle}{\langle S, x : \neg C \sqcup D \mid U, C \sqsubseteq D^{L,x} \mid K \rangle} \text{(Unfold)}$$

$x \in \mathcal{D}(\mathbf{B})$ and $x \notin L$

$$\frac{\langle S, x : C \sqcap D \mid U \mid K \rangle}{\langle S, x : C, x : D \mid U \mid K \rangle} (\sqcap^+)$$

$$\frac{\langle S, x : \neg(C \sqcap D) \mid U \mid K \rangle}{\langle S, x : \neg C \mid U \mid K \rangle \quad \langle S, x : \neg D \mid U \mid K \rangle} (\sqcap^-)$$

$$\frac{\langle S, x : \mathbf{T}(C) \mid U \mid K \rangle}{\langle S, x : C, x : \Box\neg C \mid U \mid K \rangle} (\mathbf{T}^+)$$

$$\frac{\langle S, x : \neg\mathbf{T}(C) \mid U \mid K \rangle}{\langle S, x : \neg C \mid U \mid K \rangle \quad \langle S, x : \neg\Box\neg C \mid U \mid K \rangle} (\mathbf{T}^-)$$

$$\frac{\langle S \mid U \mid K \rangle}{\langle S, x : \Box\neg C \mid U \mid K \rangle \quad \langle S, x : \neg\Box\neg C \mid U \mid K \rangle} (cut)$$

if $x : \neg\Box\neg C \notin S$ and $x : \Box\neg C \notin S$

$x \in \mathcal{D}(\mathbf{B})$ $\quad C \in \mathcal{L}_{\mathbf{T}}$

$$\frac{\langle S, u : \exists R.C \mid U \mid K \rangle}{\langle S, u \xrightarrow{R} y_1, y_1 : C \mid U \mid K \rangle \quad \cdots \quad \langle S, u \xrightarrow{R} y_m, y_m : C \mid U \mid K \rangle} (\exists^+)$$

if $\mathcal{D}(\mathbf{B}) = \{y_1, \ldots, y_m\}$

$$\frac{\langle S, u : \neg\Box\neg C_1, \ldots, u : \neg\Box\neg C_n \mid U \mid K, u : \neg\Box\neg C_1, \ldots, u : \neg\Box\neg C_n \rangle}{\langle S, y_1 : C_k, y_1 : \Box\neg C_k, S^M_{u \to y_1}, \overline{S}^{\Box^{-k}}_{u \to y_1} \mid U \mid K \rangle \cdots \langle S, y_m : C_k, y_m : \Box\neg C_k, S^M_{u \to y_m}, \overline{S}^{\Box^{-k}}_{u \to y_m} \mid U \mid K \rangle} (\Box^-)$$

if $\mathcal{D}(\mathbf{B}) = \{y_1, \ldots, y_m\}$ and $y_1 \neq u, \ldots, y_m \neq u$

The Tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$

- $\langle S \mid U \mid \emptyset \rangle$ is the corresponding constraint system of KB
- F= query
- S'= set of constraints obtained by adding to S the constraint corresponding to $\neg F$
- The calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ checks whether a query F is minimally entailed from a KB by means of the following procedure:
  - (phase 1) the calculus $\mathcal{TAB}_{PH1}^{\mathcal{EL}^{\perp}\mathbf{T}}$ is applied to $\langle S' \mid U \mid \emptyset \rangle$;
  - if, for each branch **B** built by $\mathcal{TAB}_{PH1}^{\mathcal{EL}^{\perp}\mathbf{T}}$, either
    - (i) **B** is closed or
    - (ii) (phase 2) the tableau built by the calculus $\mathcal{TAB}_{PH2}^{\mathcal{EL}^{\perp}\mathbf{T}}$ for $\langle S \mid U \mid \mathbf{B}^{\Box^-} \rangle$ is open,

    then KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$, otherwise KB $\not\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$.

An example

- $\{\exists hc.S(j), \mathbf{T}(S) \sqsubseteq NTP\} \models_{\mathcal{EL}^{\perp}\mathbf{T}_{\textsf{min}}} \exists hc.NTP(j)$

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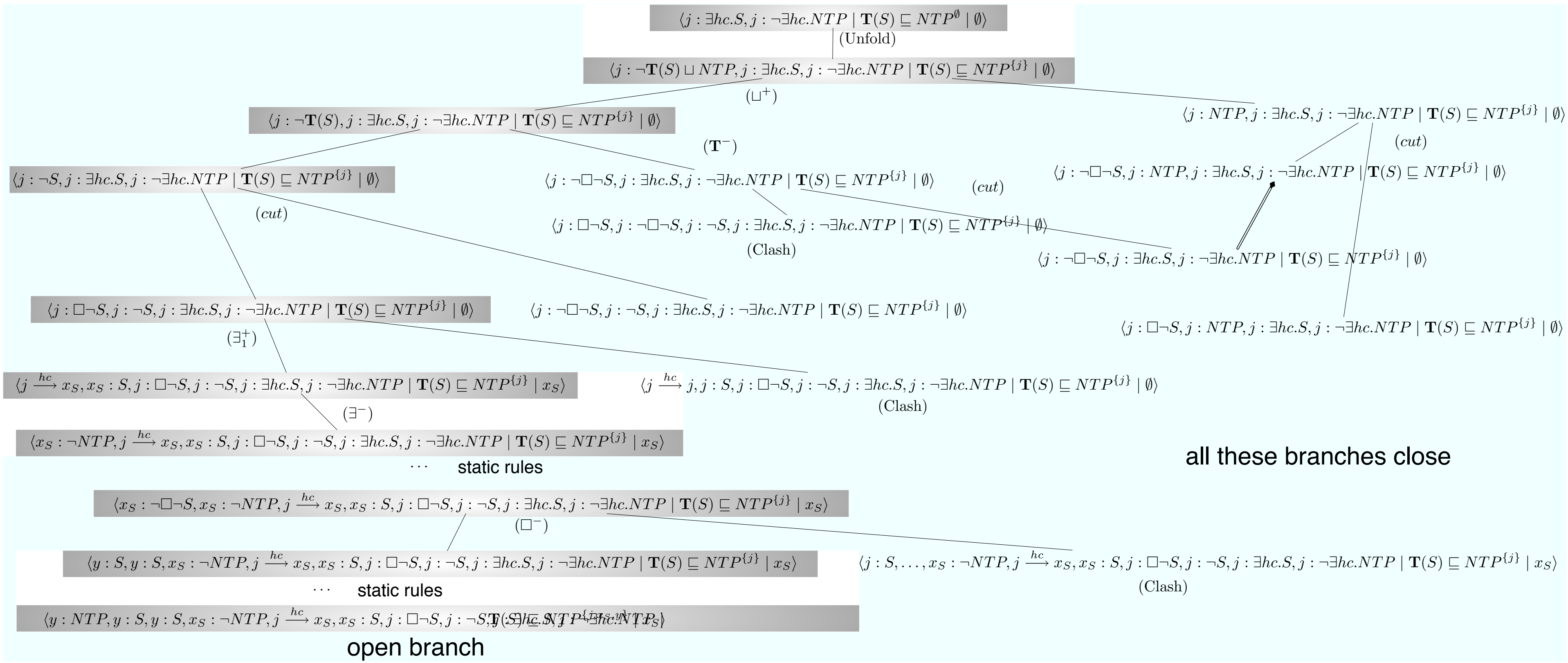

An example

$\mathcal{D}(\mathbf{B}) = \{j, y, x_S\}$

$$\langle j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\emptyset} \mid x_S : \neg\Box\neg S \rangle$$

(Unfold)

$$\langle j : \neg\mathbf{T}(S) \sqcup NTP, j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

$(\sqcup^+)$

$$\langle j : \neg\mathbf{T}(S), j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

$$\langle j : NTP, j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

...

$(\mathbf{T}^-)$

$$\langle j : \neg S, j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

$$\langle j : \neg\Box\neg S, j : \exists hc.S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

$(\text{Clash})_{\Box^-}$

$(\exists^+)$

$$\langle j \xrightarrow{hc} y, y : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

...

$$\langle j \xrightarrow{hc} j, j : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

(Clash)

$$\langle j \xrightarrow{hc} x_S, x_S : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j\}} \mid x_S : \neg\Box\neg S \rangle$$

(Unfold)

$$\langle x_S : \neg\mathbf{T}(S) \sqcup NTP, j \xrightarrow{hc} x_S, x_S : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j, x_S\}} \mid x_S : \neg\Box\neg S \rangle$$

$(\sqcup^+)$

$$\langle x_S : \neg\mathbf{T}(S), j \xrightarrow{hc} x_S, x_S : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j, x_S\}} \mid x_S : \neg\Box\neg S \rangle$$

...

$$\langle x_S : NTP, j \xrightarrow{hc} x_S, x_S : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j, x_S\}} \mid x_S : \neg\Box\neg S \rangle$$

...

(cut) and static rules

...

$$\langle y : \Box\neg S, j : \Box\neg S, x_S : \Box\neg S, x_S : NTP, j \xrightarrow{hc} x_S, x_S : S, j : \neg S \mid \mathbf{T}(S) \sqsubseteq NTP^{\{j, x_S, y\}} \mid x_S : \neg\Box\neg S \rangle$$

complete open branch

The Tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$

- **Theorem**: $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ is a sound and complete decision procedure for verifying if KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$.
- **Proposition**: Given a KB and a query F, the problem of checking whether KB $\cup\{\neg F\}$ is satisfiable is in NP.
- **Theorem**: The problem of deciding whether KB $\models_{\mathcal{EL}^{\perp}\mathbf{T}_{\text{min}}} F$ by means of $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ is in Π_2^p. (matching known complexity)

Conclusions

- We have provided a two-phase tableau calculus $\mathcal{TAB}_{min}^{\mathcal{EL}^{\perp}\mathbf{T}}$ for minimal entailment in the Left Local fragment of the logic $\mathcal{EL}^{\perp}\mathbf{T}_{\mathrm{min}}$ of the family of low complexity DLs $\mathcal{EL}^{\perp}$.
- The proposed calculus matches the known complexity results: Π_2^p
- A similar tableau procedure can be defined for DL-lite$_c\mathbf{T}$ fragment for which a Π_2^p upper bound for minimal entailment has been shown [GGOP].
- Study optimizations.
- Find polynomial fragments for minimal entailment, in analogy with circumscription [PFS10].

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Thank you!!!