On a Logic for Coalitional Games with Priced-Resource Agents

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Outline

1. Introduction

2. The logic *Priced* RB-ATL (PRB-ATL)
   - Model checking
   - Optimization problem

3. Conclusions
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Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players.
- Agents can join in coalitions/teams to collectively perform tasks/reach goals.
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**Two sides of the same coin**

*Artificial Intelligence/Game theory*
Agents and coalitions

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Two sides of the same coin
Artificial Intelligence/Game theory

Logical Formalisms
Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)
CL and ATL

CL and ATL


CL and ATL


Theorem (Goranko, TARK 2001)

CL can be embedded into ATL
Addition of bounds on resources to ATL

World is small (resources are bounded)
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Extensions of ATL with bounds on resources:
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Extensions of ATL with bounds on resources:

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2\cdot r+1} \times |G|)$

No lower bound
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No lower bound

RAL [Bulling, Farwer, ECAI 2010]

If actions may produce resources, then Model Checking becomes UNDECIDABLE
Resource Bounded ATL (RB-ATL)

Team $A$

Endowment: $\eta : A \rightarrow \mathbb{N}^r$

$\langle \langle A^\eta \rangle \rangle \diamond p$ whatever other agents do
Due to the nesting of the team operators in a formula, the agents can be provided with a new endowment of resources to perform subtasks.

$$\langle \langle A^n \rangle \rangle \ominus \langle \langle A^{n'} \rangle \rangle \diamond p$$

agents of team A, equipped with the endowment of resources $\eta$, can force the next state to be s.t. they can guarantee that $p$ eventually holds equipped with the new endowment $\eta'$.
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D. Della Monica
Motivations

- In the current approaches there is not a notion of global availability of resources
  - new endowment for each subtask
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- Very significant present-day issues related to procurement of resources:
  - resources are available on the market (or in nature) in limited amount
  - the cost for achieving them depends on such an availability (price of resources)
Our contributions

1. We introduce the **global availability of resources on the market**
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased

2. We introduce the notion of **price of resources**
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)
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Our contributions - cont’d

3 Model checking
   ▶ PSPACE-hardness
   ▶ Recover decidability even if actions produce resources
      ★ actions may produce a resource in a quantity that is not greater than
         the amount that has already been consumed so far
      ★ the global availability of the market will never be greater than the
         initial global availability
      ★ several significant real-world scenarios fit (e.g., acquiring memory by
         a program, leasing a car during a travel)

4 Optimization problem
   ▶ minimization of the amount of money needed to acquire the
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Syntax and semantics

Formulae of PRB-ATL are given by the grammar:

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \langle\langle \vec{A}\$\rangle\rangle \diamond \varphi \mid \langle\langle \vec{A}\$\rangle\rangle \varphi U \varphi \mid \langle\langle \vec{A}\$\rangle\rangle \Box \varphi \]
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Formulae of PRB-ATL are evaluated wrt:

- a priced game structure $G$
- a location $q$ of $G$
- an initial availability of resources $\vec{m}$
A priced game structure $G$ is a weighted graph:
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- **locations** are labeled by **atomic propositions** (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of **actions** to be performed
- any possible combination of actions gives rise to **transitions** (edges of the graph)
- **actions** consume and produce **resources**
- each resource has a **price** that is variable and depends on the current availability of that resource on the market, the location $q$ of $G$ and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough **money** to acquire them
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PSPACE-hardness

Theorem (Della Monica, Napoli, Parente, \textit{submitted to LAMAS 2011})

The model checking problem for PRB-ATL is PSPACE-hard

Reduction from the \textit{TQBF} problem
\textit{(the problem of determining whether a Fully Quantified Boolean Formula is true)}

\textbf{Fully Quantified Boolean Formula} a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier
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Parametric PRB-ATL formulae

- **PRB-ATL:**
  \[ \varphi = \langle\langle A^1 \rangle\rangle \diamond (\langle\langle A^2 \rangle\rangle \bigcirc p \lor \langle\langle A^3 \rangle\rangle q \bigcup p) \]

**Definition (Cost of a PRB-ATL formula)**

\[ f_{\text{cost}}(\varphi) = A_1 \cdot \$_1 + A_2 \cdot \$_2 + A_3 \cdot \$_3 \]

- **parametric PRB-ATL:**
  \[ \varphi_{\vec{X}} = \langle\langle X^1 \rangle\rangle \diamond (\langle\langle X^2 \rangle\rangle \bigcirc p \lor \langle\langle A^3 \rangle\rangle q \bigcup p) \]
The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine coalitions that satisfy a PRB-ATL formula with minimum cost

[Della Monica, Napoli, Parente, *submitted to LAMAS 2011*]
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A brute force algorithm only requires polynomial space
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**Theorem**

*The Optimal Coalition problem is PSPACE-complete*
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Conclusions and future work

Conclusions:

- PRB-ATL: a formalism to model scenarios with bounded, priced resources
  - Model checking PRB-ATL is PSPACE-complete
  - Determine the optimal coalitions formation is PSPACE-complete
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- PRB-ATL: a formalism to model scenarios with bounded, priced resources
  - Model checking PRB-ATL is PSPACE-complete
  - Determine the optimal coalitions formation is PSPACE-complete

Future work:
- To study variants of the logic (e.g., agents can be viewed as resources)
- Resource-bounded extensions of other classical formalisms (e.g., \( \mu \)-calculus)