On a Logic for Coalitional Games with Priced-Resource Agents

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CILC 2011
August 31, 2011 - Pescara

Outline

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Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players
- Agents can join in coalitions/teams to collectively perform tasks/reach goals

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Artificial Intelligence/Game theory

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

CL and ATL

CL [Pauly, Journal of Logic and Computation, 2002]

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Theorem (Goranko, TARK 2001)

CL *can be embedded into* ATL

Addition of bounds on resources to ATL

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Extensions of ATL **with bounds on resources:**

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Extensions of ATL **with bounds on resources:**

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$

No lower bound

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RAL [Bulling, Farwer, ECAI 2010]

If actions may produce resources,
then Model Checking becomes UNDECIDABLE

Resource Bounded ATL (RB-ATL)

Team A

Endowment: $\eta : A \to \mathbb{N}^r$

$\langle\langle A^\eta \rangle\rangle \Diamond p$ whatever other agents do

A crucial property

Due to the nesting of the team operators in a formula, the agents can be provided with a new endowment of resources to perform subtasks

$\langle\langle A^{\eta}\rangle\rangle \bigcirc \langle\langle A^{\eta'}\rangle\rangle \Diamond p$ agents of team A, equipped with the endowment of resources η, can force the next state to be s.t. they can guarantee that p eventually holds equipped with the new endowment η'

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  - new endowment for each subtask UNREALISTIC
- Very significant present-day issues related to procurement of resources:
  - resources are available on the market (or in nature) in limited amount
  - the cost for achieving them depends on such an availability (price of resources)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

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Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - ⋆ actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - ⋆ the global availability of the market will never be greater than the initial global availability
     - ⋆ several significant real-world scenarios fit (e.g., acquiring memory by a program, leasing a car during a travel)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

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Syntax and semantics

Formulae of PRB-ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \bigcirc \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \varphi \mathcal{U} \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \Box \varphi$$

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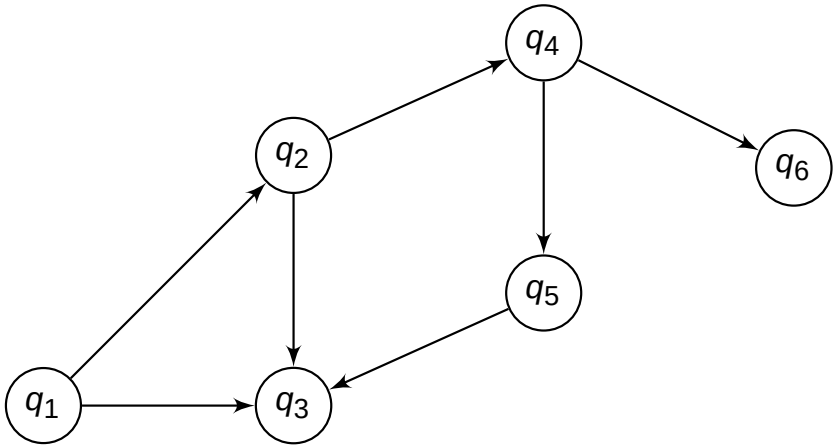
Formulae of PRB-ATL are evaluated wrt:

- a priced game structure G
- a location q of G
- an initial availability of resources $\vec{m}$

Priced game structure

A priced game structure G is a weighted graph:



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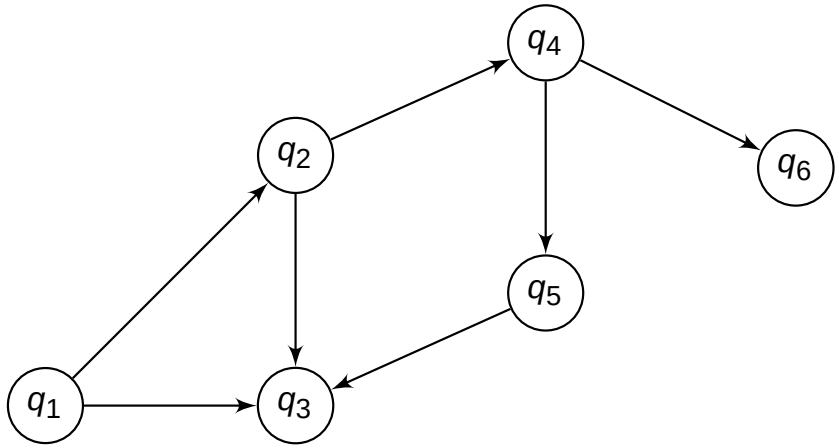

- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

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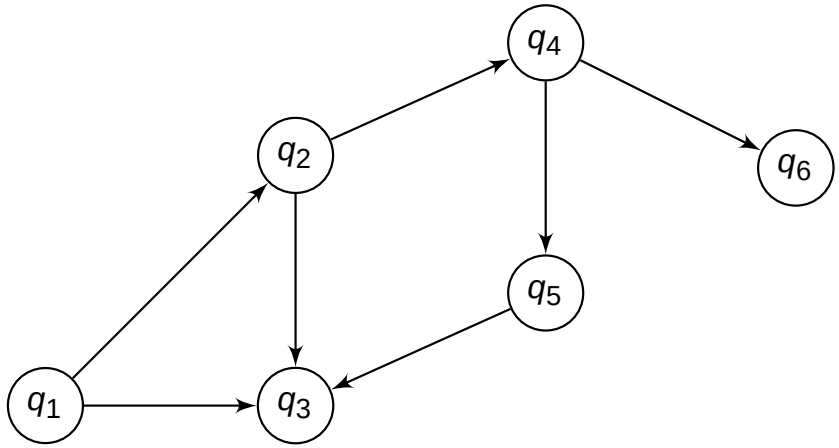

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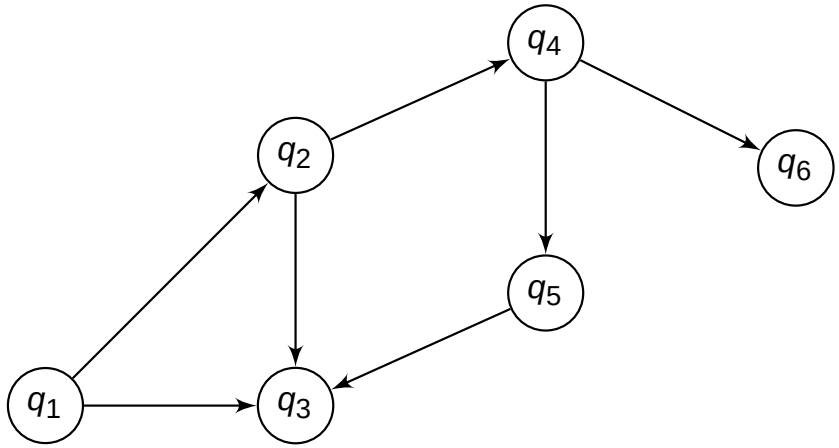

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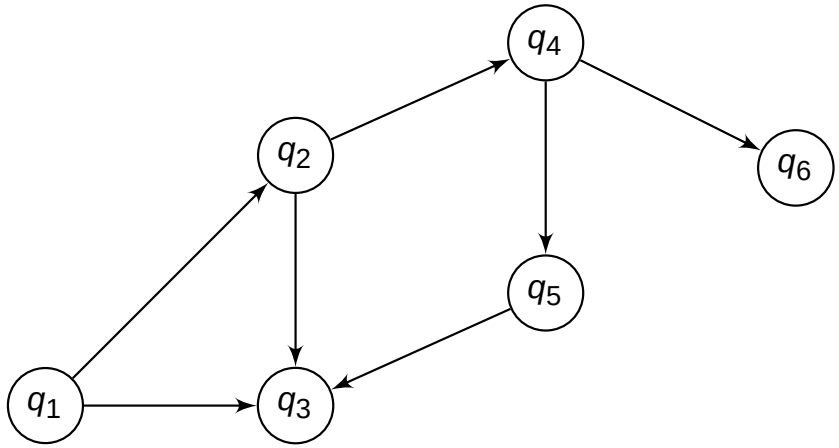

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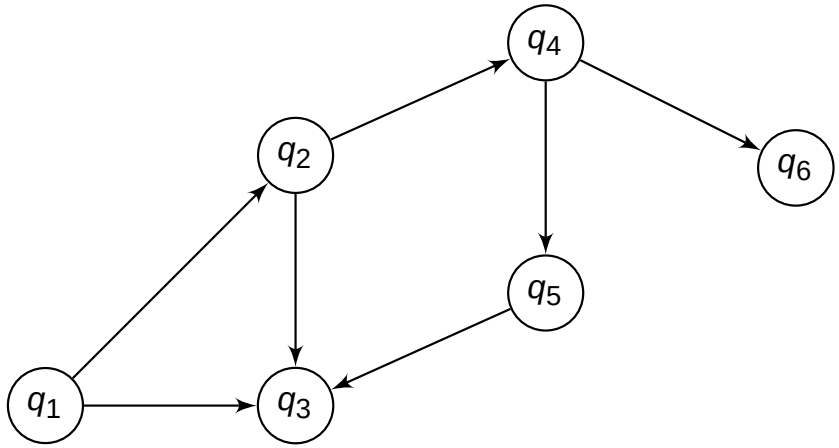

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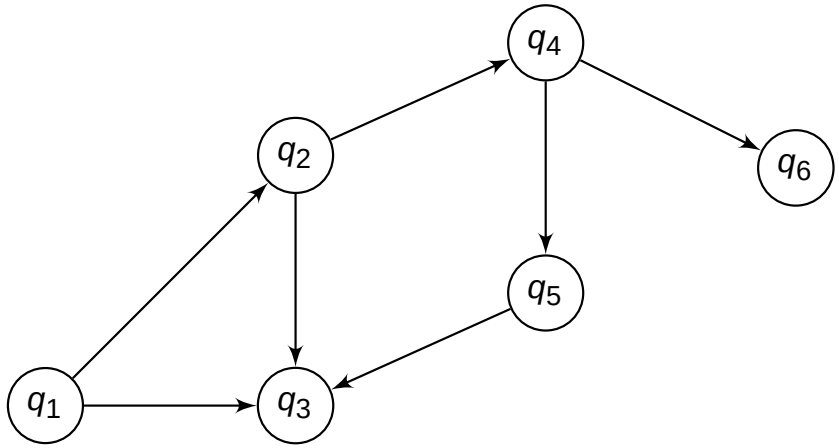

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PSPACE-hardness

Theorem (Della Monica, Napoli, Parente, *submitted to LAMAS 2011*)

The model checking problem for PRB-ATL *is PSPACE-hard*

Reduction from the *TQBF* problem

(the problem of determining whether a *Fully Quantified Boolean Formula* is true)

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

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Parametric PRB-ATL formulae

- PRB-ATL: $\varphi = \langle\langle A_1^{\vec{\$}_1}\rangle\rangle\Diamond(\langle\langle A_2^{\vec{\$}_2}\rangle\rangle \bigcirc p \vee \langle\langle A_3^{\vec{\$}_3}\rangle\rangle q \mathcal{U} p)$

Definition (Cost of a PRB-ATL formula)

$$f\_cost(\varphi) = A_1 \cdot \vec{\$}_1 + A_2 \cdot \vec{\$}_2 + A_3 \cdot \vec{\$}_3$$

- parametric PRB-ATL: $\varphi_{\vec{X}} = \langle\langle X_1^{\vec{\$}_1}\rangle\rangle\Diamond(\langle\langle X_2^{\vec{\$}_2}\rangle\rangle \bigcirc p \vee \langle\langle A_3^{\vec{\$}_3}\rangle\rangle q \mathcal{U} p)$

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine coalitions that satisfy a PRB-ATL formula with minimum cost

[Della Monica, Napoli, Parente, *submitted to LAMAS 2011*]

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Theorem

The Optimal Coalition problem is PSPACE-complete

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Conclusions and future work

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- PRB-ATL: a formalism to model scenarios with bounded, priced resources
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  - Determine the optimal coalitions formation is PSPACE-complete

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Future work:

- To study variants of the logic (e.g., agents can be viewed as resources)
- Resource-bounded extensions of other classical formalisms (e.g., μ-calculus)