Cyclic pregroups and natural language: a computational algebraic analysis

C. Casadio and M. Sadrzadeh

♣Dip. Studi Classici, Univ. Gabriele D'Annunzio, Chieti, Italy

♠Dept. of Computer Science, Oxford University, UK

CILC - 26-esimo Convegno Italiano di Logica Computazionale

31 Agosto - 2 Settembre 2011, Pescara

INTRODUCTION

In the paper we study the application of cyclic logical rules to the analysis of word order and movement phenomena in natural languages. The need of some kind of cyclic operations or transformations was envisaged both by **Z. Harris** [1966, 1968] and **N. Chomsky** [e.g. 1981, 1986].

We present a formal approach to natural languages based on the calculus of Pregroups developed by **J. Lambek** [cf. 1999, 2004, 2008] implemented with certain cyclic rules derived from Noncommutative Multiplicative-Additive (Cylic) Linear Logic (NMALL, CyMALL) studied by V. M. Abrusci [cf. 2002].

The formal system so obtained gives an efficient grammar for the computation of strings of words as grammatical sentences belonging to different kinds of natural languages: from Persian to Romance languages, such as Italian and French, and German languages, such as Dutch.

1. CYCLIC RULES IN THEORETICAL LINGUISTICS

Harris developed a cyclic cancellation automaton [1966, 1968] as the simplest device to recognize sentence structure by computing words strings through cancellations of a given symbol with its left (or right) inverse.

The formalism proposed by Harris is sufficient for many languages, requiring just string concatenation for sentence derivation, but the same limitations of context free grammars are met since just adjacent constituents are processed. Different kinds of cyclic transformations were explored by Chomsky for computing long distance dependencies [e.g. 1986].

Lambek argues in [2008] that his calculus of Pregroups meets the requirements of Chomsky's transformational grammar expressing **movement traces** by means of **double adjoints**.

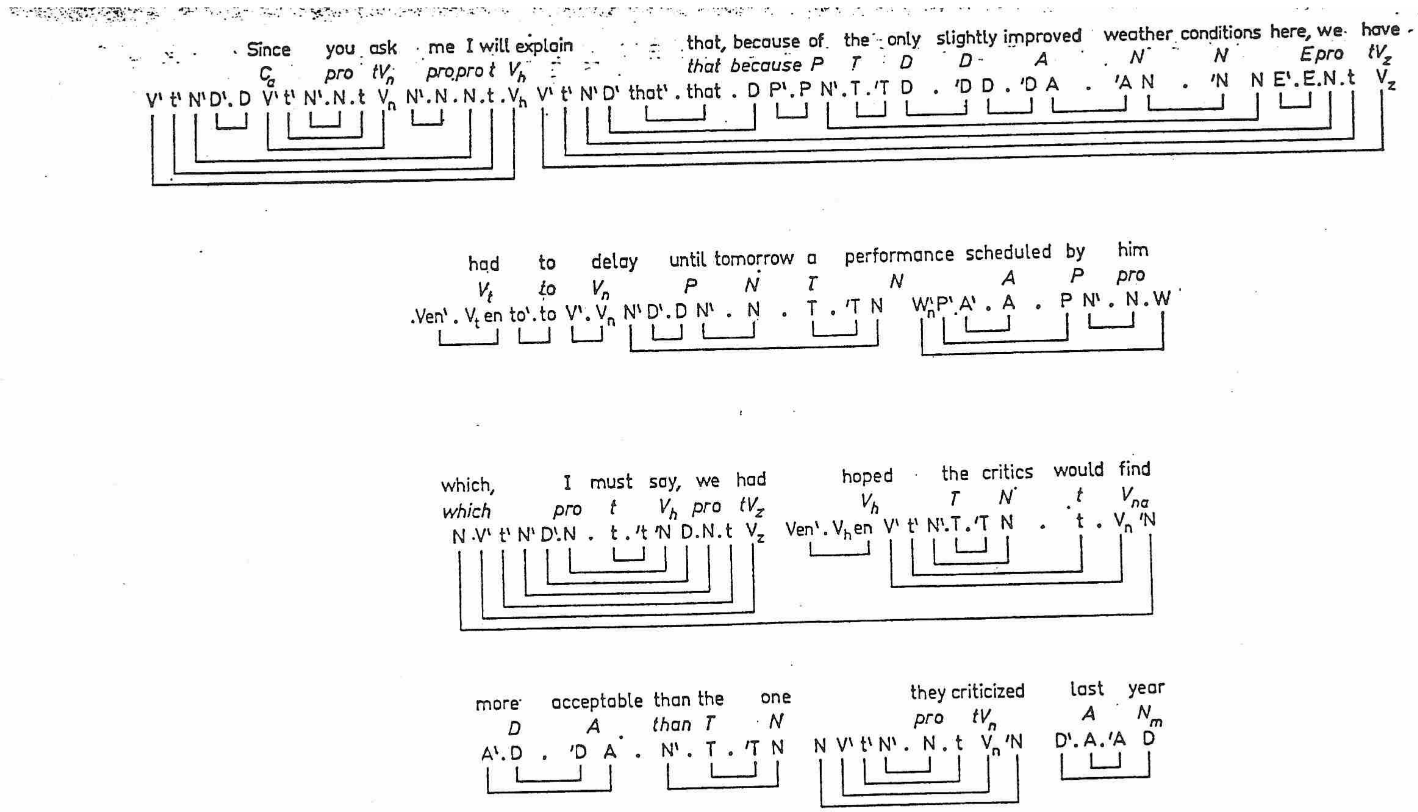

Since you ask me I will explain that, because of the only slightly improved weather conditions here, we have
had to delay until tomorrow a performance scheduled by him
which, I must say, we had hoped the critics would find
more acceptable than the one they criticized last year

Fig. 1.

2. Cyclic Logic

We refer to logical cyclicity as the property of logical systems developed from cyclic linear logic [cf. Yetter 1990]. Of particular interest for linguistic analysis is the system of Noncommutative (Cyclic) Multiplicative Linear Logic (NMLL), discussed in Abrusci [2002]. This system is directly connected to the system of Classical Bilinear Logic studied by Lambek, from which the calculus of Pregroups has been developed.

Pregroups are non conservative extensions of NMLL in which *left* ${}^{\perp}a$ and *right* $a^{\perp}$ negations are defined as *left* a^{ℓ} and *right* a^{r} adjoints.

Abrusci, M.: Classical Conservative Extensions of Lambek Calculus. Studia Logica, 71, 277–314 (2002)

Lambek, J.: From Word to Sentence. A Computational Algebraic Approach to Grammar. Polimetrica, Monza (MI) (2008)

3. Pregroup Grammar

A *pregroup* $\{G, \cdot, 1, {}^{\ell}, {}^{r}, \rightarrow\}$ is a partially ordered monoid in which each element *a* has a *left adjoint* a^{ℓ}, and a *right adjoint* a^{r} such that

$$a^{\ell} a \rightarrow 1 \rightarrow a\, a^{\ell}$$

$$a\, a^{r} \rightarrow 1 \rightarrow a^{r} a$$

where the dot "**.**", that is usually omitted, is the monoid operation with unit 1, and the arrow denotes the partial order. In linguistic applications the symbol 1 denotes the empty string of types and the monoid operation is interpreted as concatenation. Adjoints are unique and it is proved that

$$1^{\ell} = 1 = 1^{r},$$

$$(a \cdot b)^{\ell} = b^{\ell} \cdot a^{\ell} \quad , \quad (a \cdot b)^{r} = b^{r} \cdot a^{r} \, ,$$

$$\frac{a \rightarrow b}{b^{\ell} \rightarrow a^{\ell}} \quad , \quad \frac{a \rightarrow b}{b^{r} \rightarrow a^{r}} \quad , \quad \frac{b^{\ell} \rightarrow a^{\ell}}{a^{\ell\ell} \rightarrow b^{\ell\ell}} \quad , \quad \frac{b^{r} \rightarrow a^{r}}{a^{rr} \rightarrow b^{rr}} \quad .$$

Linguistic applications make particular use of the equation $a^{r\ell} = a = a^{\ell r}$, allowing the cancellation of double opposite adjoints, and of the rules

$$a^{\ell\ell}\, a^{\ell} \rightarrow 1 \rightarrow a^{\ell}\, a^{\ell\ell} \, , \qquad a^{r}\, a^{rr} \rightarrow 1 \rightarrow a^{rr}\, a^{r}$$

contracting and expanding identical *left* and *right* double adjoints respectively. Just the *contractions*

$$a^{\ell}\, a \rightarrow 1 \;\text{ and }\; a\, a^{r} \rightarrow 1$$

are needed to determine that a string of words is a sentence.

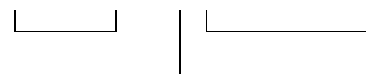
A pregroup is *freely generated* by a partially ordered set of *basic* types. From each basic type *a* we form *simple* types by taking single or repeated adjoints: $\ldots$ $a^{\ell\ell}$, a^{ℓ}, a, a^{r}, $a^{rr} \ldots$. *Compound* types or just *types* are strings of simple types.

Like in categorial grammars we have two essential steps: (i) assign one or more (basic or compound) types to each word in the dictionary; (ii) check the grammaticality and sentencehood of a string of words by a calculation on the corresponding types, where the only rules involved are *contractions* and *ordering postulates* such as $\alpha \to \beta$ (α, β basic types).

In the Pregroup we have **basic types** $n, \pi, o, \omega, \lambda, s$, **simple types** such as $n^l, n^r, \pi^l, \pi^r, o^l, o^r$, and **compound types** such as $(\pi^r s\, o^l)$. For example, the types of the constituents of the sentence “I saw him.” are as follows

I saw him.

$$\pi \quad (\pi^r s\, o^l) \quad o$$



We say that a sentence is grammatical iff it *reduces* to the type s, a procedure depicted by the under-link diagrams.

Lambek J., A computational algebraic approach to English grammar, *Syntax* 7:2, 128-147, 2004.

Lambek J., From word to sentence: a pregroup analysis of the object pronoun who(m), *Journal of Logic, Language and Information* 16, 302-323, 2007.

Casadio C. and J. Lambek (eds.), *Recent computational algebraic approaches to morphology and syntax*, Polimetrica, Milan, 2008.

(*Maria is wondering whom do you say to have met*)

Maria si chiede chi dici che hai incontrato

$$\pi_3 \, (\pi_3^r \, \mathsf{s}_1 \, \sigma^\ell) \, (\overline{q} \, \hat{\mathsf{o}}^{\ell\ell} q^\ell) \, (q_1 \, (\pi_2^\ell) \, \sigma^\ell) \, (\overline{\mathsf{s}} \, \tilde{\mathsf{s}}^\ell) \, (\mathsf{s}_1 \, \mathsf{o}^\ell) \to \mathsf{s}_1$$

$$= \pi_3 \, (\pi_3^r \, \mathsf{s}_1 \, \sigma^\ell) \, (\overline{q} \, \hat{\mathsf{o}}^{\ell\ell} q^\ell) \, (q_1 \, \sigma^\ell) \, (\overline{\mathsf{s}} \, \tilde{\mathsf{s}}^\ell) \, (\mathsf{s}_1 \, \mathsf{o}^\ell) \to \mathsf{s}_1$$

4. CROSS LINGUISTIC MOTIVATIONS

In Persian the subject and object of a sentence occur in pre-verbal position (Persian is a SOV language), but they may attach themselves as clitic pronouns to the end of the verb and form a **one-word sentence**. By doing so, the word order changes from SOV to VSO.

A similar phenomenon (but in the other directiob) happens in languages like Italian and French: verbal complements occurring in post-verbal position, can take a clitic form and move to a pre-verbal position.

These movements have been accounted in Pregroup grammar for French and Italian by means of double adjoints (Bargelli, Casadio, Lambek). In this paper we take a different approach offering a unified account of clitic movement by adding two **cyclic rules** (or meta-rules) to the lexicon of the Pregroup grammar. The import of these rules is that the **clitic type** of the verb is derivable from its **original type**.

4.1 Embedding cyclic rules into Pregroups

We extend the Pregroup calculus with two cyclic rules that allow to analyse a variety of movement phenomena in natural languages.

IMPORTANT: the addition of cyclic rules is not equivalent to the reintroduction of the structural rule of *Commutativity* into the pregroup calculus (a logic without structural rules like the Syntactic Calculus).

These rules are derivable into NMLL (or also CyMLL) cf. Abrusci [2002]

$$\frac{\vdash \Gamma, \Delta}{\vdash \Delta^{+2}, \Gamma}(rr) \qquad \frac{\vdash \Gamma, \Delta}{\vdash \Delta, \Gamma^{-2}}(ll)$$

In the notation of Pregroups ($positive$ formulae as $right$ adjoints and $negative$ formulae as $left$ adjoints), the formulation of the two cyclic rules becomes

$$(1)\ qp \leq pq^{ll} \qquad (2)\ qp \leq p^{rr}q$$

The monoid multiplication of the pregroup is **non commutative**, but if we add to a pregroup the following **meta-rules**, then we obtain a limited form of commutativity, for $p, q \in P$

Clitic Rule (1): If $p^r q$ is the *original* type of the verb, then so is $q\overline{p}^l$.

Clitic Rule (2): If qp^l is the *original* type of the verb, then so is $\overline{p}^r q$.

The over-lined types $\overline{p}^l$, $\overline{p}^r$ are introducend as a notational convenience to distinguish the clitic pronouns from the non-clitic stressed pronouns or arguments.

For any clitic pronoun p, we postulate the partial order $\overline{p} \leq p$ to express the fact that a clitic pronoun is also a kind of pronoun. We assume that for all $p, q \in P$, we have $\overline{pq} = \overline{p}\,\overline{q}$.

5.1 Clitic Movement in Persian

In Persian the subject and object of a sentence occur in pre-verbal position (Persian is a SOV language), but they may attach themselves as clitic pronouns to the end of the verb and form a one-word sentence (word order changes from SOV to VSO).

The clitic clusters (pre-verbal vs. post-verbal) for the sentence *I saw him*, “man u-ra didam” in Persian, exhibit the following general pattern:

I	him	saw	saw	I	him
man	u-ra	didam.	did	am	ash.
π	o	$(o^r \pi^r s)$	$s\overline{o}^l\overline{\pi}^l$	$\overline{\pi}$	$\overline{o}$

The over-lined types $\overline{\pi}, \overline{o}$, stand for the clitic versions of the subject and object pronouns.

Including **clitic rule** (1) in the lexicon of the pregroup grammar of Persian, we obtain the clitic form of the verb from its original type.

The original Persian verb has the type

$o^r\pi^r s = (\pi o)^r s$ which is of the form $p^r q$

after applying the clitic rule we obtain

$s(\overline{\pi o})^l = s(\overline{\pi}\,\overline{o})^l = s\overline{o}^l\overline{\pi}^l$

i.e. the type of the verb with postverbal clitics.

The clitic rule can be seen as a re-write rule and the derivation can be depicted as a one-liner as follows

$$o^r\pi^r s = (\pi o)^r s \quad \rightsquigarrow \quad s(\overline{\pi o})^l = s\overline{o}^l\overline{\pi}^l$$

To form these one-word sentences, one does not necessarily have pronouns for subject and object in the original sentence. They can as well be formed from sentences with nominal subjects and objects, for example the sentence *I saw Nadia*, in Persian “man Nadia-ra didam”, becomes “did-am-ash” and is typed exactly as above.

Hassan	Nadia	saw
Hassan	Nadia-ra	did.
π	o	$(o^r \pi^r s)$

saw	he	her
di	d	ash.
$s\overline{o}^l\overline{\pi}^l$	$\overline{\pi}$	$\overline{o}$

Sadrzadeh, M.: Pregroup Analysis of Persian Sentences. In Casadio and Lambek (eds.)(2008)

5.2 Clitic Movement in French

In French, the clitic clusters move in the opposite direction with respect to Persian. We need therefore the **clitic rule** (2). Using this rule we can derive the type of the clitic form of the verb from its original type. Consider a simple example, the sentence “Jean voit Marie.” (*Jean sees Marie*) and its clitic form “Jean la voit”. We type these as follows

Jean	voit	Marie.
π	$(\pi^r s\, o^l)$	o

Jean	la	voit.
π	$\overline{o}$	$(\overline{o}^r \pi^r s)$

To derive the clitic type of the verb from its original type, we start with the original type

voit : $(\pi^r s o^l)$

take $q = \pi^r s$ and $p^l = o^l$

apply the clitic rule (2) and obtain the type: $(\overline{o}^r \pi^r s)$

This is an example with the locative object λ and its clitic pronoun $\overline{\lambda}$.

Jean	va	à Paris.
π	$(\pi^r s\, \lambda^l)$	λ

Jean	y	va.
π	$\overline{\lambda}$	$(\overline{\lambda}^r \pi^r s)$

Again the clitic rule (2) easily derives $(\overline{\lambda}^r \pi^r s)$ from $(\pi^r s \lambda^l)$.

Now consider the more complicated example “Jean donne une pomme à Marie” (*Jean gives an apple to Marie*); we type it as follows

Jean	donne	une pomme	à Marie.
π	$(\pi^r s\, w^l o^l)$	o	w

While learning French at school, it’s difficulty to remember the order of the clitic pronouns in these sentences. Clitic rule (2) offers a hint: according to it a verb of the type $(\pi^r s\, w^l o^l)$ can also be of type $\overline{w}^r \overline{o}^r \pi^r s$, taking $q = (\pi^r s)$ and $p = (ow)^l$. This type will result in the following grammatical sentence

Jean	la	lui	donne.
π	$\overline{o}$	$\overline{w}$	$\overline{w}^r \overline{o}^r \pi^r s$

5.3 Clitic Movement in Italian

Sentences with one occurrence of a pre-verbal clitic can be obtained exactly like in French

Gianni	vede	Maria.
π	$(\pi^r s\, o^l)$	o

Gianni	la	vede.
π	$\overline{o}$	$(\overline{o}^r \pi^r s)$

To derive the clitic type of the verb we start with the original type $(\pi^r s o^l)$, take $q = \pi^r s$ and $p^l = o^l$, apply the clitic rule (2) and obtain the type $(\overline{o}^r \pi^r s)$. The same process applies with a locative argument λ and the corresponding clitic pronoun $\overline{\lambda}$, where the clitic rule derives $(\overline{\lambda}^r \pi^r s)$ from $(\pi^r s \lambda^l)$.

Gianni	va	a Roma.
π	$(\pi^r s\, \lambda^l)$	λ

Gianni	ci	va.
π	$\overline{\lambda}$	$(\overline{\lambda}^r \pi^r s)$

When we consider the more complicated cases of a verb with two arguments like in "Gianni da un libro a Maria" (*Gianni gives a book to Maria*), or "Gianni mette un libro sul tavolo" (*Gianni puts a book on the table*) we find that clitics pronouns occur in the opposite order with respect to French: e.g. the verb "dare" (*to give*) has the clitic form "Gianni glie lo da".

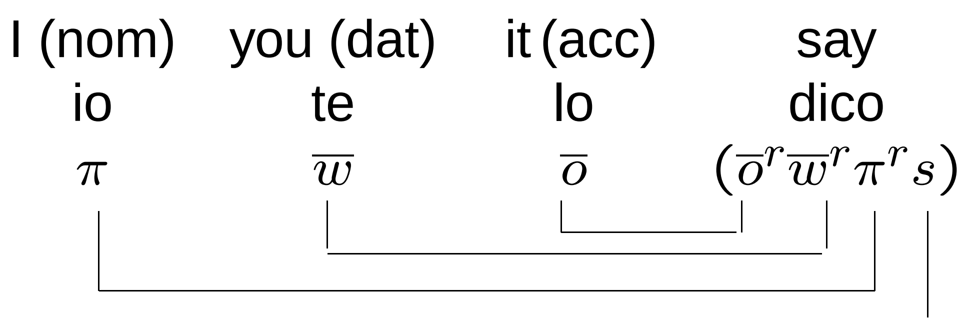
In Casadio and Lambek [2001] this problem was handled by introducing a second type for verbs with two complements $(\pi^r s\, o^l w^l)$ and $(\pi^r s\, o^l \lambda^l)$; assuming these verb types and applying clitic rule (2) we obtain the correct clitic verb forms to handle the cases of pre-verbal cliticization

$$(\pi^r s\, o^l w^l) = (\pi^r s (wo)^l) \rightsquigarrow ((\overline{wo}^r)\ \pi^r\ s) = (\overline{o}^r\ \overline{w}^r\ \pi^r\ s)$$

the same with λ in place of o.

Gianni	glie	lo	da.
π	$\overline{w}$	$\overline{o}$	$(\overline{o}^r \overline{w}^r \pi^r s)$

The following diagram shows the general pattern of preverbal cliticization in Italian with a verb taking two arguments:



6. Word order in Dutch subordinate clauses

In Dutch (like in German), the position of the finite verb in main clauses differs from that in subordinate clauses. The unmarked order of the former is SVO, while the latter exhibit an SOV pattern. Also concerning word order Dutch is similar to German in that the finite verb always occurs in second position in declarative main clauses (V2), while the verb appears in final position in subordinate clauses: a sentence like

“hij kocht het boek” (*he bought the book*)

in subordinate clauses becomes

“. . . hij het boek kocht” (*he the book bought*) with more arguments

“Jan geeft het boek aan Marie” (*Jan gives the book to Marie*)

becomes “. . . Jan het boek aan Marie geeft” (*Jan the book to Marie gives*).

In order to reason about these kinds of word order changes, we generalize our **clitic rule** (2), corresponding to the *r*ight cyclic axiom, to all words by removing the bar from the types and the word ‘original’ from the definition, obtaining the following rule allowing verb argoments to move up the string from right to left

Move Rule (1): If qp^{ℓ} is the type of the verb, so is $p^r q$.

The rule allows us to correctly type the examples mentioned above

hij	kocht	het boek	
he	bought	the book	
π	$(\pi^r s\, o^\ell)$	o	$\longrightarrow s$

omdat hij	het boek	kocht	
because he	the book	bought	
ss^ℓ π	o	$(o^r \pi^r s)$	$\longrightarrow s$

omdat Jan	het boek	aan Marie	geeft	
because Jan	the book	to Marie	gives	
ss^ℓ π	o	w	$(w^r o^r \pi^r s)$	$\longrightarrow s$

7. Some conclusive remarks

It is interesting that the rules for clitic movement correspond to logical rules of cyclicity. Accordingly, one may call French and Italian *right cyclic* languages and Persian a *left cyclic* language.

We can refer to the clitic rules (1) and (2) as *cyclic axioms*, in particular to the first one as the *left cyclic axiom* and to the second one as the *right cyclic axiom*

Persian $p^r q \leq q\overline{p}^l$ French-Italian $qp^l \leq \overline{p}^r q$

We also prove the following results:

Proposition .1 *The* clitic *axioms are derivable from the* cyclic *axioms.*

Proof. The axiom for French and Italian is derivable form the *right cyclic axiom* as follows, take p to be p^l and observe that $(p^l)^{rr} = p^r$, then one obtains $qp^l \leq p^r q$. Since $\overline{p} \leq p$, and since adjoints are contravariant, we have $p^r \leq \overline{p}^r$, thus $p^r q \leq \overline{p}^r q$, and by transitivity of order we obtain $qp^l \leq \overline{p}^r q$. The axiom for Persian is derivable from the *left cyclic axiom* as follows: take q to be p^r and p to be q. Now since $(p^r)^{ll} = p^l$, we obtain $p^r q \leq qp^l$, and since $\overline{p} \leq p$, by contravariance, $p^l \leq \overline{p}^l$, thus $qp^l \leq q\overline{p}^l$, and by transitivity of order $p^r q \leq q\overline{p}^l$.

Casadio, C., Sadrzadeh, M.: Clitic Movement in Pregroup Grammar: a Cross-linguistic Approach. Proceedings of Eighth International Tbilisi Symposium on Language, Logic and Computation, Springer 2011.

8 Insights into Hungarian and Dutch word order

Word order in Hungarian has been recently studied by Sadrzadeh [2010]

As work in progress we intend to consider a set of well known examples concerning word order, including examples such as German vs. Dutch verb final clauses.

Sadrzadeh, M.: An Adventure into Hungarian Word Order with Cyclic Pregroups. In AMS-CRM proceedings of Makkai Fest. (2010)

Bibliography

Abrusci, M.: Classical Conservative Extensions of Lambek Calculus. Studia Logica, 71, 277–314 (2002)

Buszkowski, W.: Type Logics and Pregroups. Studia Logica, 87(2/3), 145–169 (2007)

Casadio, C.: Non-Commutative Linear Logic in Linguistics. Grammars 4(3), 167-185 (2001)

Casadio, C.: Agreement and Cliticization in Italian: A Pregroup Analysis. LATA 2010, Lecture Notes in Computer Science 6031, Springer 166-177 (2010)

Casadio, C., Lambek, J. (eds.): Recent Computational Algebraic Approaches to Morphology and Syntax. Polimetrica, Milan (2008)

Casadio, C., Sadrzadeh, M.: Clitic Movement in Pregroup Grammar: a Cross-linguistic Approach. Proceedings of Eighth International Tbilisi Symposium on Language, Logic and Computation, (Bakuriani, Georgia 2009). Springer 2011.

Chomsky, N.: Lectures on Government and Binding. Dordrecht, Foris (1981)

Chomsky, N.: Barriers. Cambridge, The MIT Press (1986)

Harris, Z. S.: A Cycling Cancellation-Automaton for Sentence Well-Formedness. International Computation Centre Bulletin 5, 69–94 (1966)

Harris, Z. S.: Mathematical Structures of Language, Interscience Tracts in Pure and Applied Mathematics, John Wiley & Sons., New York (1968)

Lambek, J.: The Mathematics of Sentence Structure. American Mathematics Monthly 65, 154–169 (1958)

Lambek, J.: Type Grammar Revisited. In: Lecomte A. et al. (eds.) Logical Aspects of Computational Linguistics, LNAI vol. 1582, pp. 1–27. Springer (1999)

Lambek, J.: Type Grammar Meets German Word Order. Theoretical Linguistics 26, 19–30 (2000)

Lambek, J.: A computational Algebraic Approach to English Grammar. Syntax 7(2), 128–147 (2004)

Lambek, J.: From Word to Sentence. A Computational Algebraic Approach to Grammar. Polimetrica, Monza (MI) (2008)

Morrill, G.: Categorial Grammar. Logical Syntax, Semantics, and Processing. Oxford University Press, Oxford (2010)

Moortgat, M.: Symmetric Categorial Grammar. Journal of Philosophical Logic, 38, 681–710 (2009)

Preller, A., Sadrzadeh, M.: Semantic Vector Space and Functional Models for Pregroup Grammars. Journal of Logic, Language and Information (t.a.)

Sadrzadeh, M.: Pregroup Analysis of Persian Sentences. In Casadio and Lambek (eds.)(2008)

Sadrzadeh, M.: An Adventure into Hungarian Word Order with Cyclic Pregroups. In AMS-CRM proceedings of Makkai Fest. (2010)

Yetter, D. N.: Quantales and (non-Commutative) Linear Logic. Journal of Symbolic Logic, 55 (1990)