Cyclic Pregroups and Natural Language: a Computational Algebraic Analysis

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Abstract. The calculus of pregroups is introduced by Lambek [1999] as an algebraic computational system for the grammatical analysis of natural languages. Pregroups are non commutative structures, but the syntax of natural languages shows a diffuse presence of cyclic patterns exhibited in different kinds of word order changes. The need of cyclic operations or transformations was envisaged both by Z. Harris and N. Chomsky, in the framework of generative transformational grammar. In this paper we propose an extension of the calculus of pregroups by introducing appropriate cyclic rules that will allow the grammar to formally analyze and compute word order and movement phenomena in different languages such as Persian, French, Italian, Dutch and Hungarian. This cross-linguistic analysis, although necessarily limited and not at all exhaustive, will allow the reader to grasp the essentials of a pregroup grammar, with particular reference to its straightforward way of computing linguistic information.

1 Introduction

In this paper we apply logical cyclic rules to the analysis of word order changes in natural languages. The need of some kind of cyclic operations or transformations was envisaged both by Harris [1966, 1968] and Chomsky [1981, 1986] for the treatment of the linguistic contexts referred to with the term *movement*. In the paper we present a formal approach to natural language based on two cyclic rules that hold in the systems of Noncommutative and Cyclic Multiplicative Linear Logic (NMLL,CyMLL), developed by Abrusci [1991, 2002] from Yetter [1990]. A critical move of this paper is to embed such cyclic rules into the calculus of Pregroups recently introduced by Lambek [1999, 2001, 2008]. The calculus has been succesfully applied to a variety of natural languages from English and German, to French and Italian, and others [see Casadio and Lambek 2008].

We show that the formal grammar obtained by so extending the pregroup calculus allows one to compute string of words belonging to various kinds of natural languages, deriving grammatical sentences involving different types of word order changes or *movements*, with particular reference to the way in which unstressed clitic pronouns attach to their verbal heads. Cross-linguistic evidence is provided comparing languages belonging to the Indo-European family, like Persian, on the one side, French and Italian, on the other, as representatives of the Romance group. Moreover the analysis is extended to include Dutch, as a representative of the West Germanic group, and Hungarian, as a representative of the Uralic family, non related to the Indo-European languages. Such cross-linguistic prespective extends the results of preceding work [Casadio and Sadrzadeh 2011, Sadrzadeh 2010], and the analysis proposed for Dutch is new.

We conclude with a short discussion of the logical and methodological connections of the present analysis to cyclic linear logic [Yetter 1990, Abrusci 1991, 2002].

2 Cyclic rules for the Calculus of Pregroups

2.1 Pregroup grammar

Pregroups are introduced by Lambek in [1999] as an alternative to the Syntactic Calculus, a well known model of categorial grammar largely applied in the fields of theoretical and computational linguistics; see e.g. Moortgat [1997], Morrill [2010]. The calculus of pregroups is a particular kind of substructural logic that is compact and non-commutative [Buszkowsi 2001, 2007]. Pregroups in fact are non conservative extensions of Noncommutative Multiplicative Linear Logic (NMLL) in which *left* and *right* iterated negations, equivalently *left* and *right* iterated adjoints, do not cancel [Abrusci 2001, Casadio 2001, Casadio and Lambek 2002, Lambek 2001, 2008].

A pregroup $\{G, \cdot, 1, {}^{\ell}, {}^{r}, \rightarrow\}$ is a partially ordered monoid in which each element a has a *left adjoint* a^{ℓ}, and a *right adjoint* a^{r} such that

$$a^{\ell} a \rightarrow 1 \rightarrow a\, a^{\ell}$$
$$a\, a^{r} \rightarrow 1 \rightarrow a^{r} a$$

where the dot "$\cdot$", that is usually omitted, stands for multiplication with unit 1, and the arrow denotes the partial order[1]. In linguistic applications syntactic types (or categories) are assigned to the words in the dictionary of a language, the symbol 1 is assigned to the empty string of types, and the operation of multiplication is interpreted as linguistic concatenation. Adjoints are unique and the following results are proved (see Lambek [2008] for details)

$$1^{\ell} = 1 = 1^{r}\,,$$
$$(a \cdot b)^{\ell} \;=\; b^{\ell} \cdot a^{\ell} \quad , \quad (a \cdot b)^{r} \;=\; b^{r} \cdot a^{r}\;,$$
$$\frac{a \rightarrow b}{b^{\ell} \rightarrow a^{\ell}} \quad , \quad \frac{a \rightarrow b}{b^{r} \rightarrow a^{r}} \quad , \quad \frac{b^{\ell} \rightarrow a^{\ell}}{a^{\ell\ell} \rightarrow b^{\ell\ell}} \quad , \quad \frac{b^{r} \rightarrow a^{r}}{a^{rr} \rightarrow b^{rr}} \quad .$$

[1] A partial order '$\leq$' (here denoted by the arrow '$\rightarrow$') is a binary relation which is reflexive: $x \leq x$, transitive: $x \leq y$ and $y \leq z$ implies $x \leq z$, and anti-symmetric: $x \leq y$ and $y \leq x$ implies $x = y$. We may read $x \leq y$ as saying that everything of type x is also of type y. The arrow is introduced to show the inference between types, like in type logical grammmars.

Linguistic applications make particular use of the equation $a^{r\ell} = a = a^{\ell r}$, allowing the cancellation of double opposite adjoints, and of the rules

$$a^{\ell\ell}\ a^{\ell} \to 1 \to a^{\ell}\ a^{\ell\ell} \quad , \quad a^{r}\ a^{rr} \to 1 \to a^{rr}\ a^{r}$$

contracting and expanding *left* and *right* adjoints respectively; just *contractions* are needed to check and determine if a given string of words is a sentence:

$$a^{\ell}\, a \to 1 \ \text{ and } \ a\, a^{r} \to 1\ .$$

A pregroup is *freely generated* by a partially ordered set of *basic* types. From each basic type a we form *simple* types by taking single or repeated adjoints: $\ldots\ a^{\ell\ell}$, a^{ℓ}, a, a^{r}, $a^{rr}\ldots$. *Compound* types or just *types* are strings of simple types.

Like in categorial grammars we have two essential steps: (i) assign one or more (basic or compound) types to each word in the dictionary; (ii) check the grammaticality and sentencehood of a string of words by a calculation on the corresponding types, where the only rules involved are *contractions* and *ordering postulates* such as $\alpha \to \beta$ (α, β basic types).

Taking as basic types: n (noun), π (nominative argument), o (accusative argument), w (dative argument), λ (locative argument), i (infinitive verb), s (sentence), we obtain *simple types* such as n^{ℓ}, n^{r}, π^{ℓ}, π^{r}, o^{ℓ}, o^{r}, $\ldots$, and *compound types* such as $(\pi^{r} s\, o^{\ell})$, the type of a transitive verb with subject in the nominative case and object in the accusative case. For example, the types of the constituents of the sentence "I saw him." are as follows, where the subscript 1 in π_1 means first person singular, and the subscript 2 in s_2 indicates the past tense[2]

I saw him.
$\pi_1\ \ (\pi_1^{r} s_2\, o^{\ell})\ \ o$

We say that a sentence is grammatical iff the computation (or calculation) of the types assigned to its words *reduces* to the type s, a procedure depicted by the under-link diagrams[3].

2.2 Cyclic rules in theoretical linguistics

In the Sixties Zellig Harris developed a cyclic cancellation automaton [1966, 1968] as the simplest device to recognize sentence structure by computing strings of words through cancellations of a given symbol with its left (or right) inverse.

[2] We analyze a sentence of the form SUBJ VP by assigning types $(\pi_k\ s_j)$, for $j = 1, \ldots, 7$ denoting the seven basic *tenses*, and k denoting the six verbal persons (singular $k = 1, 2, 3$, plural $k = 4, 5, 6$).

[3] These diagrams are reminescent of the planar poof nets of non-commutative linear logic, connecting the formulas, decorated by a left or right adjoint with their positive counterparts, by means of under-links that satisfy the requirements of parallelism and planarity (Abrusci 2002, Lambek 1999, 2008, Buszkowski 2007).

The formalism proposed by Harris is sufficient for many languages, requiring just string concatenation for sentence derivation, but the same limitations of context free grammars are met [Francez and Kaminski 2007, Buszkowski and Moroz 2008]. Different kinds of cyclic transformations were explored by Chomsky [e.g. 1981] to compute constituents movement in long distance dependencies. As argued by Lambek [2008], the analysis of modern European languages requires that word symbols (logical types) take double superscripts, like in Harris [1968], or the double adjoints defined in pregroup grammar, wherever Chomsky's approach postulates a trace. The calculus of pregroups meets in this sense the requirements of Chomsky's transformational grammar expressing *traces* by means of *double adjoints*.

2.3 Introducing cyclic rules into pregroups

We extend the pregroup calculus with two cyclic rules that will allow us to analyse a variety of movement phenomena in natural languages. It is important to point out that the addition of cyclic rules is not equivalent to the reintroduction of the structural rule of *Commutativity* into the pregroup calculus (a logic without structural rules like the Syntactic Calculus).

These rules are derivable into NMLL (or also CyMLL) cf. Abrusci [2002]

$$\frac{\vdash \Gamma, \Delta}{\vdash \Delta^{+2}, \Gamma}(rr) \qquad\qquad \frac{\vdash \Gamma, \Delta}{\vdash \Delta, \Gamma^{-2}}(\ell\ell)$$

In the notation of pregroups (*positive* formulae as *right* adjoints and *negative* formulae as *left* adjoints), the formulation of the two cyclic rules becomes

$$(1)\ \ qp \leq p^{rr}q \qquad\qquad (2)\ \ qp \leq pq^{\ell\ell}$$

The monoid multiplication of the pregroup is *n*on-commutative, but if we add to the pregroup calculus the cyclic rules defined above as metarules, then we obtain a limited form of commutativity, for $p, q \in P$.

Metarules are postulates introduced into the dictionary of the grammar to simplify lexical assignments and make syntactic calculations quicker: the types assigned to the words of a given language are assumed to be stored permanently in the speaker's 'mental' dictionary; to prevent overloading this mental dictionary, the grammar includes metarules asserting that, if the dictionary assigns a certain type to a word, then this word may also have certain other types. The effect of the two cyclic metarules is that the *cyclic* type of each verb form is derivable from its original type.

3 Word Order and Cyclicity in Natural Languages

In the following section we present a cross-linguistic analysis comparing languages belonging to the Indo-European family, like Persian, on the one side, French and Italian, on the other side, as representatives of the Romance group.

The analysis is also extended to include Dutch, as a representative of the West Germanic group, and Hungarian, as a representative of the Uralic family, which is not related to the Indo-European family.

3.1 Cross-linguistic motivations

In Persian the subject and object of a sentence occur in pre-verbal position (Persian is a SOV language), but they may attach themselves as clitic pronouns to the end of the verb and form a one-word sentence. By doing so, the word order changes from SOV to VSO. A similar phenomenon happens in Romance languages like Italian and French, but the movement goes in the opposite direction: verbal complements occurring in post-verbal position, can take a clitic form and move to a pre-verbal position.

These movements have been accounted for in the pregroup grammar for French [Bargelli and Lambek 2001] and Italian [Casadio and Lambek 2001] by assigning clitic words types with double adjoints. In this paper we present a different approach offering a unified account of clitic movement by adding two *c*yclic rules (or metarules) to the lexicon of the pregroup grammar. The import of these rules is that the *c*litic type of the verb is derivable from its *o*riginal type.

Clitic Rule (1): If $p^r q$ is the *original* type of the verb, then so is $q\overline{p}^\ell$.

Clitic Rule (2): If qp^ℓ is the *original* type of the verb, then so is $\overline{p}^r q$.

The over-lined types $\overline{p}^\ell$, $\overline{p}^r$ are introducend as a notational convenience to distinguish the clitic pronouns from the non-clitic stressed pronouns or arguments. For any clitic pronoun p, we postulate the partial order $\overline{p} \leq p$ to express the fact that a clitic pronoun is also a kind of pronoun. We assume that for all $p, q \in P$, we have $\overline{pq} = \overline{p}\ \overline{q}$.

3.2 Clitic movement in Persian

In Persian the subject and object of a sentence occur in pre-verbal position (Persian is a SOV language), but they may attach themselves as clitic pronouns to the end of the verb and form a one-word sentence (word order changes from SOV to VSO). The clitic clusters (pre-verbal vs. post-verbal) for the sentence *I saw him*, "man u-ra didam" in Persian, exhibit the following general pattern:

I	him	saw		saw	I	him
man	u-ra	didam.		did	am	ash.
π	o	$(o^r \pi^r s)$		$s\,\overline{o}^\ell \overline{\pi}^\ell$	$\overline{\pi}$	$\overline{o}$

The over-lined types $\overline{\pi}, \overline{o}$, stand for the clitic versions of the subject and object pronouns.

Including **clitic rule** (1) in the lexicon of the pregroup grammar of Persian, we obtain the clitic form of the verb from its original type. The original Persian verb has the type: $o^r \pi^r s = (\pi o)^r s$, which is of the form $p^r q$; after applying the clitic rule we obtain: $s(\overline{\pi o})^\ell = s(\overline{\pi}\,\overline{o})^\ell = s\overline{o}^\ell\overline{\pi}^\ell$, i.e. the type of the verb with postverbal clitics. The clitic rule can be seen as a re-write rule and the derivation can be depicted as a one-liner as follows

$$o^r \pi^r s = (\pi o)^r s \quad \rightsquigarrow \quad s(\overline{\pi o})^\ell = s\overline{o}^\ell\overline{\pi}^\ell$$

To form these one-word sentences, one does not necessarily have pronouns for subject and object in the original sentence. They can as well be formed from sentences with nominal subjects and objects, for example the sentence *I saw Nadia*, in Persian "man Nadia-ra didam", becomes "did-am-ash" and is typed exactly as above.

Hassan	Nadia	saw		saw	he	her	
Hassan	Nadia-ra	did.		di	d	ash.	
π	o	$(o^r \pi^r s)$	$\rightarrow$ s	$(s\ \overline{o}^\ell\ \overline{\pi}^\ell)$	$\overline{\pi}$	$\overline{o}$	$\rightarrow$ s

One can form a yes-no question from any of the sentences above, by adding the question form "aya" to the beginning of the sentence. Since in Persian the word order of the question form is the same as that of the original sentence, the clitic movement remains the same and obeys the same rule [Sadrzadeh 2008]

Did	Hassan	Nadia	see?		Did	see	he	her?	
aya	Hassan	Nadia-ra	did?		aya	di	d	ash?	
qs^ℓ	π	o	$(o^r \pi^r s)$	$\rightarrow$ q	qs^ℓ	$(s\overline{o}^\ell\overline{\pi}^\ell)$	$\overline{\pi}$	$\overline{o}$	$\rightarrow$ q

3.3 Clitic movement in French

In French, the clitic clusters move in the opposite direction with respect to Persian. We need therefore the **clitic rule** (2). Using this rule we can derive the type of the clitic form of the verb from its original type. Consider a simple example, the sentence "Jean voit Marie." (*Jean sees Marie*) and its clitic form "Jean la voit". We type these as follows

Jean	voit	Marie.		Jean	la	voit.	
π	$(\pi^r s\, o^\ell)$	o	$\rightarrow$ s	π	$\overline{o}$	$(\overline{o}^r \pi^r s)$	$\rightarrow$ s

To derive the clitic type of the verb from its original type, we start with the original type of "voit" : $(\pi^r s\, o^\ell)$ take $q = (\pi^r s)$ and $p^\ell = o^\ell$, apply **clitic rule** (2) and obtain the type: $(\overline{o}^r \pi^r s)$. The following is an example with the locative object λ and its clitic pronoun $\overline{\lambda}$.

Jean	va	à Paris.		Jean	y	va.	
π	$(\pi^r s \lambda^\ell)$	λ	$\rightarrow$ s	π	$\overline{\lambda}$	$(\overline{\lambda}^r \pi^r s)$	$\rightarrow$ s

Again the **clitic rule** (2) easily derives $(\overline{\lambda}^r \pi^r s)$ from $(\pi^r s \lambda^\ell)$. Now consider the more complicated example "Jean donne une pomme à Marie" (*Jean gives an apple to Marie*); we type it as follows

Jean	donne	une pomme	à Marie.
π	$(\pi^r s\, w^\ell o^\ell)$	o	w

While learning French at school, it's difficulty to remember the order of the clitic pronouns in these sentences; **clitic rule** (2) offers a hint: according to it a verb of the type $(\pi^r s\, w^\ell o^\ell)$ can also be of type $\overline{w}^r \overline{o}^r \pi^r s$, taking $q = (\pi^r s)$ and $p = (ow)^\ell$. This type will result in the following grammatical sentence

Jean	la	lui	donne.
π	$\overline{o}$	$\overline{w}$	$\overline{w}^r \overline{o}^r \pi^r s$

But it will *not* make the following incorrect order grammatical

Jean	lui	la	donne.
π	$\overline{w}$	$\overline{o}$	$\overline{w}^r \overline{o}^r \pi^r s$.

3.4 Clitic movement in Italian

Sentences with one occurrence of a pre-verbal clitic can be obtained exactly like in French, as shown in the following examples corresponding to the French sentences given above: "Gianni vede Maria" and its clitic form "Gianni la vede"

Gianni	vede	Maria.		Gianni	la	vede.
π	$(\pi^r s\, o^\ell)$	o	$\rightarrow$ s	π	$\overline{o}$	$(\overline{o}^r \pi^r s) \rightarrow$ s

To derive the clitic type of the verb we start with the original type $(\pi^r s o^\ell)$, take $q = \pi^r s$ and $p^\ell = o^\ell$, apply **clitic rule** (2) and obtain the type $(\overline{o}^r \pi^r s)$. The same process is obtained with a locative argument λ and the corresponding clitic pronoun $\overline{\lambda}$, where the clitic rule derives $(\overline{\lambda}^r \pi^r s)$ from $(\pi^r s \lambda^\ell)$.

Gianni	va	a Roma.		Gianni	ci	va.
π	$(\pi^r s\, \lambda^\ell)$	λ	$\rightarrow$ s	π	$\overline{\lambda}$	$(\overline{\lambda}^r \pi^r s) \rightarrow$ s

When we consider the more complicated cases of a verb with two arguments like in "Gianni da un libro a Maria" (*Gianni gives a book to Maria*), or "Gianni mette un libro sul tavolo" (*Gianni puts a book on the table*), we find that clitics pronouns occur in the opposite order with respect to French: e.g. the verb "dare" (*to give*) has the clitic form "Gianni glie lo da" (*Gianni to-her it gives*).

In Casadio and Lambek [2001] this problem was handled by introducing a second type for verbs with two complements $(\pi^r s\, o^\ell w^\ell)$ and $(\pi^r s\, o^\ell \lambda^\ell)$; assuming these verb types and applying **clitic rule** (2) we obtain the correct clitic verb forms to handle the cases of pre-verbal cliticization:

$$(\pi^r s\, o^\ell w^\ell) = (\pi^r s (wo)^\ell) \rightsquigarrow ((\overline{wo})^r\ \pi^r\ s) = (\overline{o}^r\ \overline{w}^r\ \pi^r\ s)$$

the same with λ in place of o.

Gianni	glie	lo	da.	Gianni	ce	lo	mette.
π	$\overline{w}$	$\overline{o}$	$(\overline{o}^r \overline{w}^r \pi^r s) \rightarrow$ s	π	$\overline{\lambda}$	$\overline{o}$	$(\overline{o}^r \overline{\lambda}^r \pi^r s) \rightarrow$ s

The following diagram shows the general pattern of preverbal cliticization in Italian with a verb taking two arguments:

I (nom)	you (dat)	it (acc)	say
io	te	lo	dico
π	$\overline{w}$	$\overline{o}$	$(\overline{o}^r \overline{w}^r \pi^r s)$

4 Insights into Hungarian and Dutch word order

In the previous section we have dealt with a special kind of movement: the clitic movement, limited to certain words moving from before to after the verb (or the other way around) and becoming clitics. In this section we show that similar cyclic rules can be used to reason about movement of words in general. This movement is more free: firstly all words, or relevant words strings, can move; secondly the movement is not restricted to the context surrounding the verb.

4.1 Word order in Dutch subordinate clauses

In Dutch (like in German), the position of the finite verb in main clauses differs from that in subordinate clauses. The unmarked order of the former is SVO, while the latter exhibit an SOV pattern. Also concerning word order Dutch is similar to German in that the finite verb always occurs in second position in declarative main clauses (V2), while the verb appears in final position in subordinate clauses: a sentence like "hij kocht het boek" (*he bought the book*) in subordinate clauses becomes "… hij het boek kocht" (*he the book bought*); with more arguments, "Jan geeft het boek aan Marie" (*Jan gives the book to Marie*) becomes "… Jan het boek aan Marie geeft" (*Jan the book to Marie gives*).

In order to reason about these kinds of movement, we generalize our **clitic rule** (2), corresponding to the *r*ight cyclic axiom, to all words by removing the bar from the types and the word 'original' from the definition, obtaining the following rule allowing verb argoments to move up the string from right to left

Move Rule (1): If qp^ℓ is the type of the verb, so is $p^r q$.

The rule allows us to correctly type the examples mentioned above

hij	kocht	het boek	
he	bought	the book	
π	$(\pi^r s\, o^\ell)$	o	$\to s$

omdat	hij	het boek	kocht	
because	he	the book	bought	
ss^ℓ	π	o	$(o^r \pi^r s)$	$\to s$

omdat	Jan	het boek	aan Marie	geeft
because	Jan	the book	to Marie	gives
	π	o	w	$(w^r o^r \pi^r s)$

Consider now an example with a modal verb "Ann wil Marie kussen" (*Ann wants to kiss Marie*) and the corresponding embedded clause "dat Ann Marie wil Kussen" (*that Ann wants to kiss Marie*)

Ann	wil	Marie	kussen		dat	Ann	Marie	(wil kussen)	
Ann	wants	Marie	kiss		that	Ann	Marie	(wants kiss)	
π	$(\pi^r s\ i^\ell)$	o	$(o^r i) \to s$		ss^ℓ	π	o	$(o^r \pi^r s)$	$\to s$

By contraction we obtain the type of the string "wil kussen": $(\pi^r s\ i^\ell)\ (i\, o^\ell) \to (\pi^r s\, o^\ell)$; then by applying **move rule** (1) we obtain the type $(o^r \pi^r s)$ expecting the object to occur before the verb string. The clause-final verb clusters in Dutch and German have been estensively studied in different linguistic theories, see Steedman [1985], Haegeman and van Riemsdijk [1986], Moortgat [1997], Lambek [2000]: a common observation is that while German prefers nested dependencies, between verbs and their arguments, Dutch prefers crossed dependencies. Consider the following sentences where "geld": NP_2 and "Marie": NP_3 are arguments of "geven": V_2, "Piet": NP_1 is an argument of the perception verb "zag": V_1. In the second example, an embedded clause, the dependencies between the two verbs and their arguments are crossed.

Jan	zag	Piet	geld	Marie	geven
Jan	saw	Piet	money	Marie	give
π	$(\pi^r s\, i^\ell\, o_1^\ell)$	o_1	o_2	w	$(w^r o_2^r\, i) \to$ s

...	Jan	Piet	Marie	geld	(zag	geven)
...	Jan	Piet	Marie	money	(saw	give)
	π	o_1	w	o_2	$(o_2^r\, w^r\, o_1^r \pi^r s) \to$ s	

In the first example, "Jan zag Piet geld Marie geven" (*Jan saw Piet give money to Marie*), the type $(i\, w^\ell o_2^\ell)$ of "geven" is converted by **move rule** (1) into $(w^r o_2^r\, i)$ where $o_1 =$ "Piet", $o_2 =$ "geld", $w =$ "Marie"; for $q = i$ and $p^\ell = (w^\ell o_2^\ell) = (o_2 w)^\ell$, we have $(o_2 w)^\ell \rightsquigarrow (o_2 w)^r = (w^r o_2^r)$. In the second example, first we apply **move rule** (1) to the type $(\pi^r s\, i^\ell\, o_1^\ell)$ of "zag" and obtain $(o_1^r \pi^r s\, i^\ell)$, for $p = o_1$; then we get the type of the verb string "zag geven" by contraction: $(o_1^r \pi^r s\, i^\ell)\ (i\, o_2^\ell w^\ell) \to (o_1^r \pi^r s\, o_2^\ell w^\ell)$; finally, applying again the cyclic rule, we obtain $(o_2^r\, w^r\, o_1^r \pi^r s)$, for $p^\ell = (w\, o_2)^\ell$. A similar analysis applies to the sentence "Jan Piet Marie zag laten zwemmen" (*Jan saw Piet make Marie swim*).

...	Jan	Piet	Marie	(zag laten zwemmen)
...	Jan	Piet	Marie	(saw make swim)
	π	o_1	o_2	$(o_2^r\, o_1^r\, \pi^r\, s) \to$ s

4.2 Word order changes in Hungarian

Examples of still more radical word order changes are offered by languages such as Hungarian[4] , where the movement is caused by a change of focus in the sentence. Words move within the sentence to reflect or focus on a certain meaning. For instance the following Hungarian sentence, which has no focus in it, simply means "János took two books to Péternek yesterday".

[4] agglutinative

János	tegnap	elvitt	két könyvet	Péternek.
János	yesterday	took	two books	to Péternek.

This can become as follows

János	tegnap	**két könyvet**	vitt el	Péternek.
János	yesterday	**two books**	took	to Péternek.
π	λ	o	$(o^r \lambda^r \pi^r s w^\ell)$	w

which means "Only two books were taken by János to Péternek yesterday". This is an example of a *single move*: **két könyvet** has moved from after the verb to before it. More sophisticated movements are also possible, for instance in the following sentence

Péternek	vitt el	tegnap	János	két könyvet.
To Péternek	took	yesterday	János	two books.
w	$(w^r s o^\ell \pi^\ell \lambda^\ell)$	λ	π	o

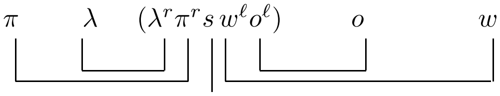
which means "It was to Péternek and to no one else that the two books were taken". This is an example of a *multi move*: not only *Péternek* has moved to the beginning of the sentence, but also first *tegnap* and then *János* have moved from before the verb to after it, and in so doing have changed their order with regard to each other. For more details on single and multi moves and a formalization of a notion of *focus*, we refer the reader to Sadrzadeh [2010]; here instead we review some examples. In order to reason about these kinds of movement, we generalize our previous cyclic rules in the following way

Move Rule (2): If $p^r q$ is $\boxed{in}$ the type of the verb, so is $q p^\ell$.

Move Rule (3): If $q p^\ell$ is $\boxed{in}$ the type of the verb, so is $p^r q$.

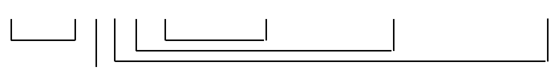
Taking π to stand for the type of the subject, o for the first object, w for the second object, and λ for the adverb, we assign the following types to the constituents of our example sentence, which had no focus in it yet

János	tegnap	elvitt	két könyvet	Péternek.
π	λ	$(\lambda^r \pi^r s\, w^\ell o^\ell)$	o	w



The focus can be on the subject or either of the objects. In each case, they will appear right before the verb after the movement. For the case of the subject, i.e. *János* the temporal adverb *yesterday* moves to after the verb, as follows

János	vitt el	tegnap	két könyvet	Péternek.
János	took	yesterday	two books	to Péternek.
π	$(\pi^r s w^\ell o^\ell \lambda^\ell)$	λ	o	w



We use our new cyclic rules to derive the new type of the verb as follows: apply **move rule** (2) to the type of the verb $(\lambda^r \pi^r s w^\ell o^\ell)$, by taking q to be $(\pi^r s\, w^\ell o^\ell)$ and p to be λ. If the focus is on the first object, i.e. *two books*, then it moves before the verb and the sentence above and its typing change as follows

János	tegnap	**két könyvet**	vitt el	Péternek.
János	yesterday	**two books**	took	to Péternek.
π	λ	o	$(o^r \lambda^r \pi^r s w^\ell)$	w

To derive the new verb type, we apply **move rule** (3) to the original type of the verb $(\lambda^r \pi^r s w^\ell o^\ell)$, by taking q to be $(\lambda^r \pi^r s w^\ell)$, and p to be o. The focus can also be on the second object *Péternek* and the verb; for details see Sadrzadeh [2010]. As an example of multi move, consider our above sentence, typed as follows

Péternek	vitt el	tegnap	János	két könyvet.
To Péternek	took	yesterday	János	two books.
w	$(w^r s\, o^\ell \pi^\ell \lambda^\ell)$	λ	π	o

Here *Péternek* has moved to the beginning of the sentence, but also first *tegnap* and then *János* have moved from before the verb to after it, and in so doing have changed their order with regard to each other. The calculation for deriving the new type of the verb reflects the above complications and needs repetitive applications of the rules. It is as follows

> Start from the original type of the verb $(\lambda^r \pi^r s w^\ell o^\ell)$ and, first *Péternek* moves to the front; to obtain this we apply rule (3) to the subtype $(\lambda^r \pi^r s w^\ell)$, take p to be w, and obtain $(w^r \lambda^r \pi^r s o^\ell)$. Then *tegnap* moves after the verb, for this we apply rule (2) to the subtype $(\lambda^r \pi^r s o^\ell)$, take p to be λ, and obtain $(w^r \pi^r s o^\ell \lambda^\ell)$. Finally *János* moves to after *tegnap*, for this we apply rule (2) to the subtype $(\pi^r s o^\ell)$, take p to be π, and obtain $(w^r s o^\ell \pi^\ell \lambda^\ell)$.

Free as they might seem, we need some restrictions to avoid certain over generations, mainly caused by the presence of the word *in*. Formulation of these exceed the purpose of this paper and can be found in Sadrzadeh [2010]. In a nutshell, they will prevent formation of types such as $(\pi^r \lambda^\ell s w^\ell o^\ell)$ and $(\lambda^r \pi^r s o^r w^\ell)$.

5 Clitic Rules and Cyclic Pregroups

Following Lambek [1999, 2001, 2008], we have formulated the clitic rules as *metarules*. At some risk of overgeneration, one is tempted to formulate these rules as axioms and add them to the pregroup calculus, or add their rule version to the sequent calculus of compact bilinear logic [Buszkowski 2001, 2002]. Note that the addition of our cyclic axioms (or cyclic rules) is not equivalent to the reintroduction of the structural rule of *Commutativity* into the pregroup calculus (a logic without structural rules like the Syntactic Calculus)[5]. These axioms belong to the *cyclic* calculus studied by Abrusci [1991, 2002] and introduced in the following way

$$\frac{\vdash \Gamma, \Delta}{\vdash \Delta^{+2}, \Gamma}(rr) \qquad\qquad \frac{\vdash \Gamma, \Delta}{\vdash \Delta, \Gamma^{-2}}(ll)$$

Via the standard translation from the Syntactic Calculus to pregroups [Lambek 1999, Buszkowski 2001] (*positive* formulae as *right* adjoints and *negative* formulae as *left* adjoints), the axiomatic version of these rules becomes

$$(1)\ qp \leq pq^{ll} \qquad\qquad (2)\ qp \leq p^{rr}q$$

We can refer to (1) and (2) as *cyclic axioms*, in particular to the first one as the *left cyclic axiom* and to the second one as the *right cyclic axiom*. We can then re-formulate our clitic *metarules* as *clitic axioms*

$$\text{Persian} \quad p^r q \leq q\overline{p}^l \qquad\qquad \text{French-Italian} \quad qp^l \leq \overline{p}^r q$$

where the latter is derivable from the former, and prove the following results:

Proposition 1. *The* clitic *axioms are derivable from the* cyclic *axioms.*

Proof. The axiom for French and Italian is derivable form the *right cyclic axiom* as follows, take p to be p^l and observe that $(p^l)^{rr} = p^r$, then one obtains $qp^l \leq p^r q$. Since $\overline{p} \leq p$, and since adjoints are contravariant, we have $p^r \leq \overline{p}^r$, thus $p^r q \leq \overline{p}^r q$, and by transitivity of order we obtain $qp^l \leq \overline{p}^r q$. The axiom for Persian is derivable from the *left cyclic axiom* as follows: take q to be p^r and p to be q. Now since $(p^r)^{ll} = p^l$, we obtain $p^r q \leq qp^l$, and since $\overline{p} \leq p$, by contravariance, $p^l \leq \overline{p}^l$, thus $qp^l \leq q\overline{p}^l$, and by transitivity of order $p^r q \leq q\overline{p}^l$.

It is interesting that the rules for clitic movement correspond to logical rules of cyclicity. Accordingly, one may call French and Italian *right cyclic* languages and Persian a *left cyclic* language. The consequences of enriching a pregroup with these cyclic axioms (or rules) are however not so desirable.

Proposition 2. *A pregroup P with either of the cyclic axioms is a partially ordered group.*

Proof. Consider the left cyclic axiom; if one takes $q = 1$, we obtain $p^r \leq p^l$ for all $p \in P$, from which one obtains $p^{ll} \leq p$. Here take $p = w^r$ for some $w \in P$ and obtain $w^l \leq w^r$. Now since we have $p^r \leq p^l$ for all p, we obtain $w^r = w^l$. A similar argument can be made for the right cyclic axiom.

[5] An approach in this line is proposed by Francez and Kaminski [2007], where a free pregroup grammar is extended by a finite set of additional (commutative) inequations between types, leading to a class of mildly context-sensitive languages, allowing the analysis of crossed dependencies and extractions.

Although, as proven by Abrusci and Lambek, cyclic bilinear logic is a conservative extension of bilinear logic (or non-commutative linear logic), this is not the case for cyclic compact bilinear logic and compact bilinear logic (the logical calculus of pregroups) [Lambek 2008, Barr 2004]. The relations among these system are however of real interest to be studied both from the logical and, particularly, from the linguistic point of view.

We conclude observing that the present analysis is consistent with previous work on French [Bargelli and Lambek 2001] and Italian [Casadio and Lambek 2001], where *iterated* adjoints are used to type clitic pronouns. We can prove in fact that iterated adjoints show up in our work too, since as observed by Lambek, the $\overline{p}^r$ used in the metarule for French and Italian is nothing but $(\overline{p}^l)^{rr}$, and the $\overline{p}^l$ used for Persian is nothing but $(\overline{p}^r)^{ll}$.

6 Conclusions

We have applied the calculus of pregroups to a selected set of sentences involving word order changes in different languages: Persian, French, Italian, Dutch and Hungarian. The cross-linguistic results we have obtained provide evidence in favour of the theoretical and computational advantages offered by the pregroup calculus extended with appropriate cyclic rules. These rules in turn represent a stimulating challenge for the development of logical grammars. We have in fact shown that those calculations, or computations, that in pregroups are dealt with logical types involving double adjoints (corresponding to Chomskian traces), can be performed, in the different languages, by means of appropriate cyclic operations.

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